

1-3 Piecewise and Step Functions

Objectives:

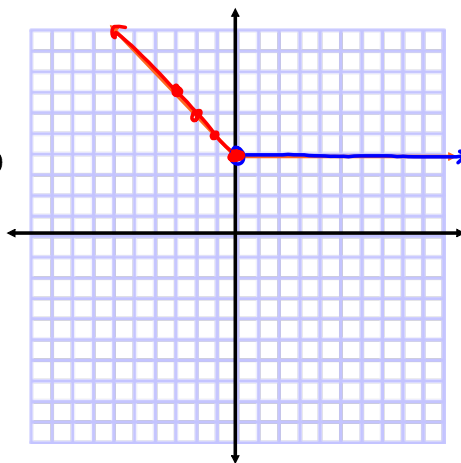
- ✱ I can graph a piecewise function
- ✱ I can write the equation of a piecewise function
- I can identify least and greatest integer functions

A piecewise function is a function with a different equations defined over unique intervals of x.

For example:

$$f(x) = \begin{cases} -x+4 & \text{if } x \leq 0 \\ 4 & \text{if } x > 0 \end{cases}$$

	x	y
$x \leq 0$	-3	7
	-2	6
	-1	5
	0	4
$x > 0$	0	4
	1	4
	2	4
	3	4

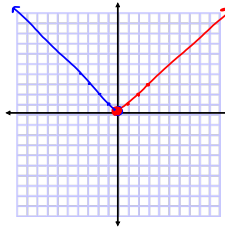


Graph the following:

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

x	y	x	y
0	0	-3	3
1	1	-2	2
2	2	-1	1
3	3	0	0

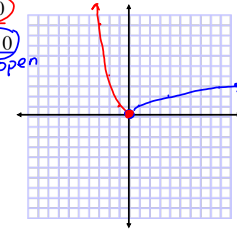
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$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$

x	y	x	y
0	0	0	0
-1	1	1	1
-2	4	4	2
-3	9	9	3

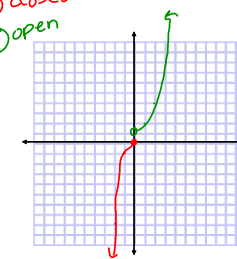
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$$f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ 2^x & \text{if } x > 0 \end{cases}$$

x	y	x	y
0	0	0	1
-1	-1	1	2
-2	-8	2	4
-3	-27	3	8
		4	16

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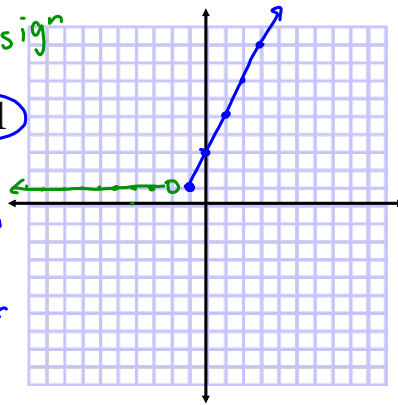
$$f(x) = \begin{cases} 1, & x < -2 \\ 2x + 3, & x \geq -1 \end{cases}$$

open

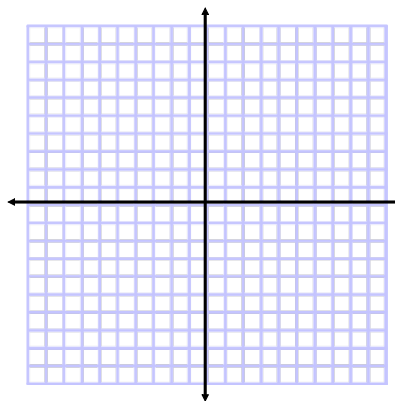
x	y
-2	1
-3	1
-4	1
-5	1

closed

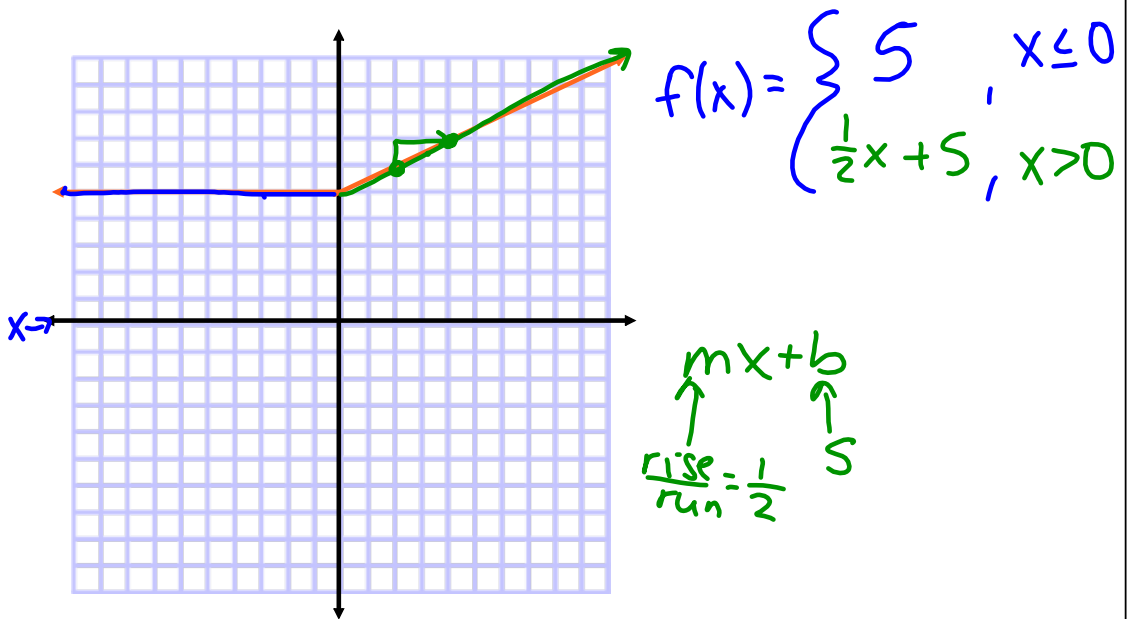
x	y
-1	1
0	3
1	5
2	7
3	9



$$f(x) = \begin{cases} x^2 - 2, & x < 0 \\ \sqrt{x}, & x > 4 \end{cases}$$



Write the equation for the following piecewise functions



Problem 2: Taxi Fares

In 2006, the rate for a taxi ride in Macon, Georgia, was \$1.20 for the first mile or part of a mile, and \$1.20 for each additional mile or part of a mile.

1. Define a piecewise function, $g(x)$, for the cost of a taxi ride up to 5 miles.

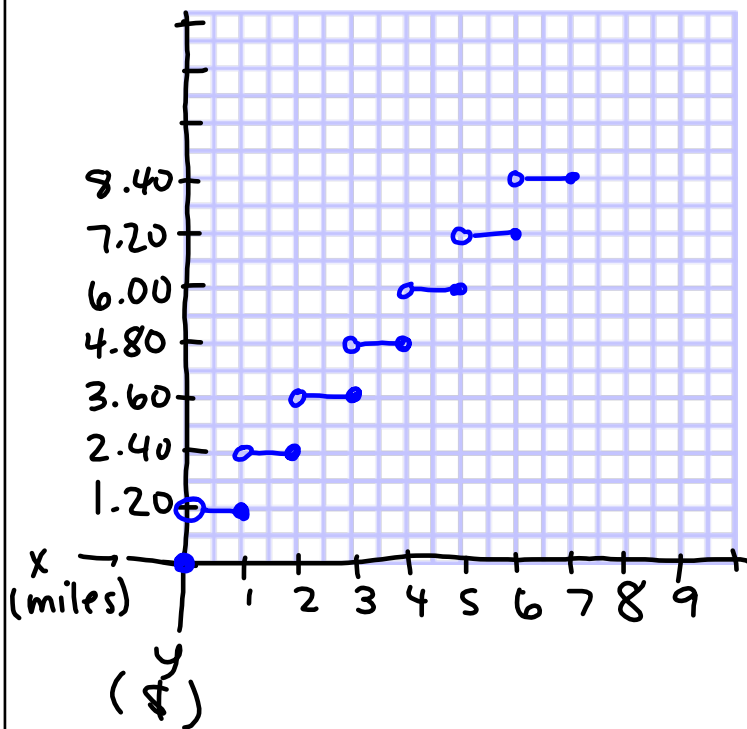
$$f(x) = \begin{cases} \$1.20, & x \leq 1 \text{ (mile)} \\ \$2.40, & 1 < x \leq 2 \\ \$3.60, & 2 < x \leq 3 \\ \$4.80, & 3 < x \leq 4 \\ \$6.00, & 4 < x \leq 5 \\ \vdots & \vdots \end{cases}$$

↑
miles

2. What is the slope of each interval? Explain your reasoning.

0 (Flat)

3. Graph $g(x)$ for $x < 3$ miles.



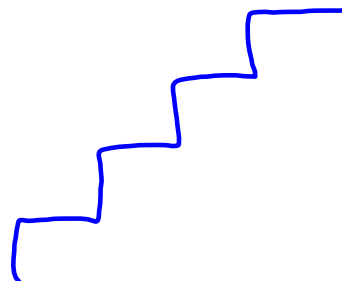
Step

4. Describe the graph of the function as either increasing or decreasing.

Neither

You have just graphed a *step function*. A **step function** is a piecewise function whose pieces are disjoint constant functions.

5. Why do you think this function is called a step function?



Problem 3 Special Step Functions

The *greatest integer function* is a special kind of step function. The **greatest integer function**, also known as the **floor function**, $G(x) = \lfloor x \rfloor$ is defined as the greatest integer less than or equal to x .

1. Evaluate each using the greatest integer function.

a. $\lfloor 2 \rfloor = \underline{2}$

b. $\lfloor 0.17 \rfloor = \underline{0}$

c. $\lfloor 2.34 \rfloor = \underline{2}$

d. $\lfloor -1.2 \rfloor = \underline{-2}$

e. $\lfloor 2.99999 \rfloor = \underline{2}$

f. $\lfloor -0.2 \rfloor = \underline{-1}$

↑
Feet

The *least integer function* is another special kind of step function. The **least integer function** $L(x) = \lceil x \rceil$ also known as the **ceiling function**, is defined as the least integer greater than or equal to x .

3. Calculate each:

a. $\lceil 2 \rceil = \underline{2}$

b. $\lceil 0.17 \rceil = \underline{1}$

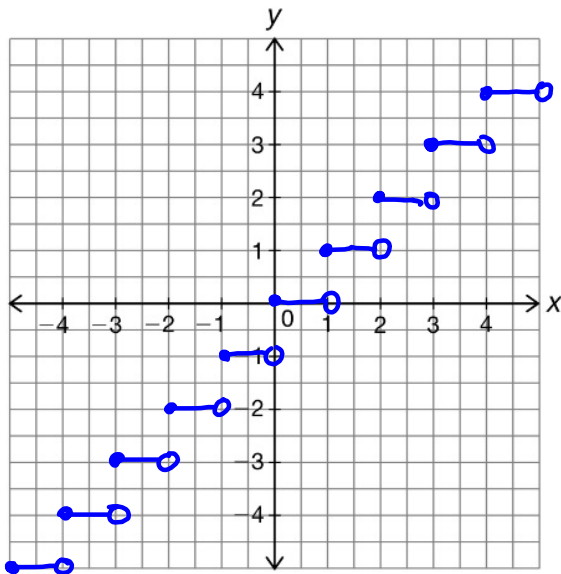
c. $\lceil 2.34 \rceil = \underline{3}$

d. $\lceil -1.2 \rceil = \underline{-1}$

e. $\lceil 2.99999 \rceil = \underline{3}$

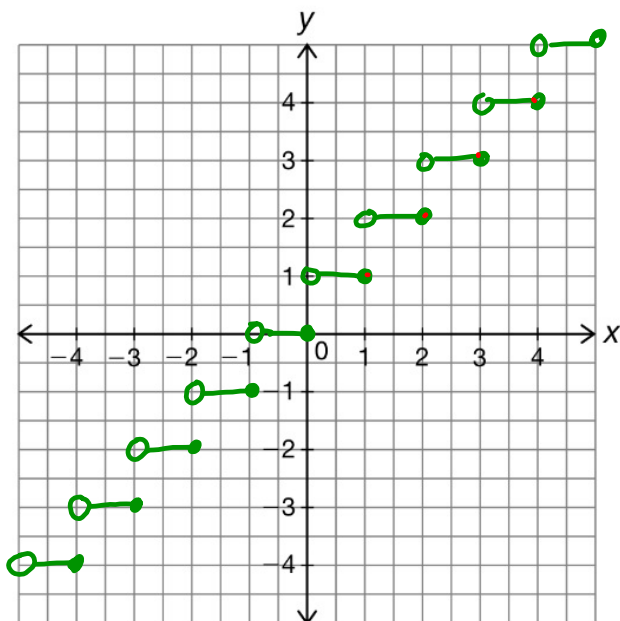
f. $\lceil -0.2 \rceil = \underline{0}$

Graph $G(x) = \lfloor x \rfloor$.



Floor

Graph $L(x) = \lceil x \rceil$.



Ceiling