1-3 Piecewise and Step Functions
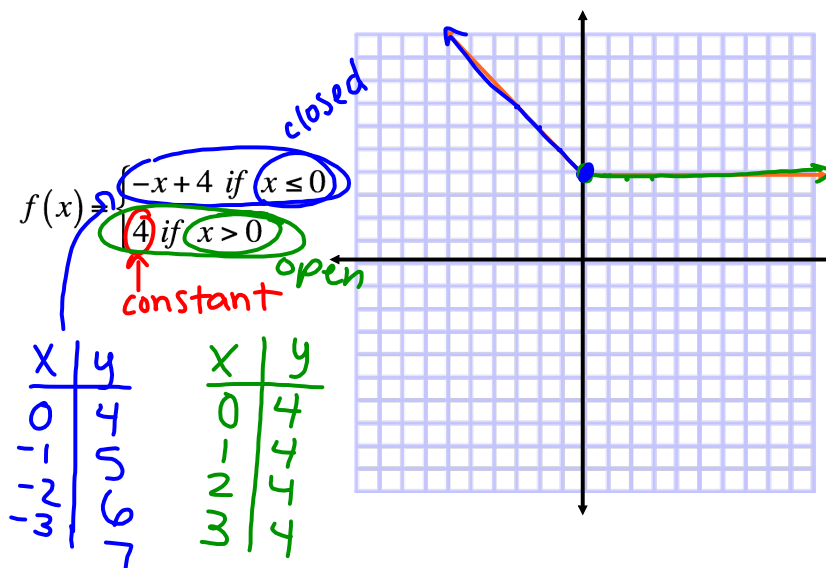
Objectives:
-I can graph a piecewise function
- I can write the equation of a piecewise function
-I can identify least and greatest integer functions

A piecewise function is a function with different equations defined over unique intervals of x.
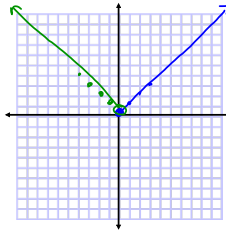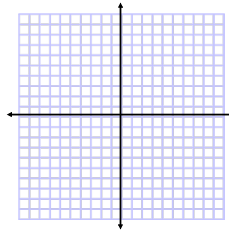
For example:

$$f(x) = \begin{cases} -x+4 & \text{if } x \le 0 \\ 4 & \text{if } x > 0 \end{cases}$$

closed

open

constant

X	y
0	4
-1	5
-2	6
-3	7

X	y
0	4
-1	4
-2	4
-3	4

Graph the following:

$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ (-x) & \text{if } x \leq 0 \end{cases}$

x	y
0	0
1	1
2	2
3	3

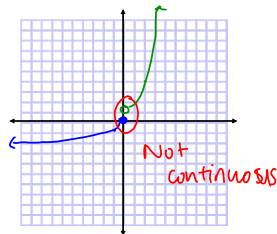| x	y
0	0
-1	1
-2	2
-3	3

$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

$f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ 2^x & \text{if } x > 0 \end{cases}$

x	y
0	0
-1	-1
-2	-8

x	y
0	1
1	2
2	4

Not continuous

$f(x) = \begin{cases} 1, & x < -2 \\ 2x+3, & x \geq -1 \end{cases}$

open

closed

x	y
-2	1
-3	1
-4	1
-5	1

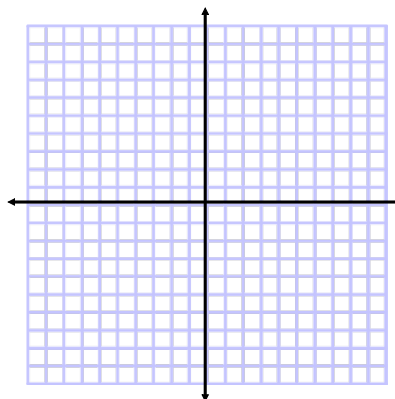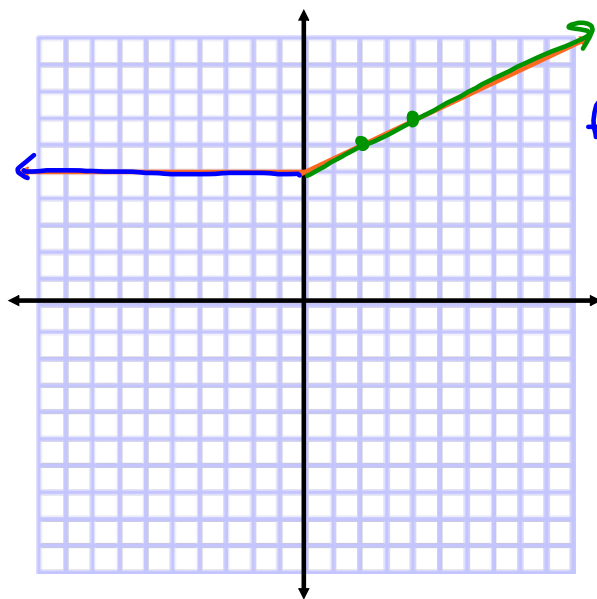| x	y
-1	1
0	3
1	5
2	7
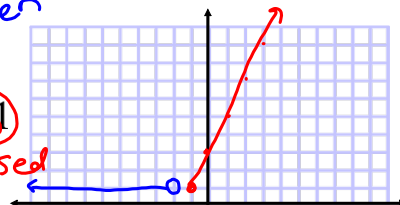
$D: (-\infty, -2) \cup [-1, \infty)$
$R: [1, \infty)$

$f(x) = \begin{cases} x^2 - 2, & x < 0 \\ \sqrt{x}, & x > 4 \end{cases}$

Write the equation for the following piecewise functions

$$f(x) = \begin{cases} 5 & , x < 0 \\ \frac{1}{2}x + 5 & , x \geq 0 \end{cases}$$

$$mx + b$$
$$\underset{rise = \frac{1}{2}}{\underset{run}{\uparrow}} \qquad \underset{5}{\uparrow}$$

Problem 2: Taxi Fares

In 2006, the rate for a taxi ride in Macon, Georgia, was $1.20 for the first mile or part of a mile, and $1.20 for each additional mile or part of a mile.
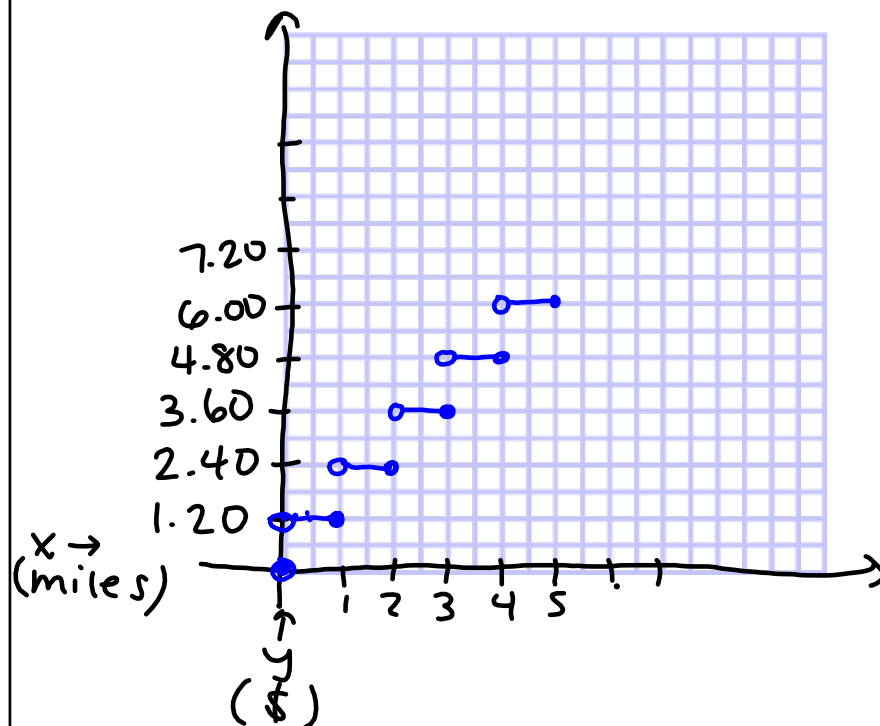
1. Define a piecewise function, g(x), for the cost of a taxi ride up to 5 miles.

$$f(x) = \begin{cases} \$1.20, & 0 < x \leq 1 \\ \$2.40, & 1 < x \leq 2 \\ \$3.60, & 2 < x \leq 3 \\ \$4.80, & 3 < x \leq 4 \\ \$6.00, & 4 < x \leq 5 \end{cases}$$

miles

2. What is the slope of each interval? Explain your reasoning.

3. Graph g(x) for x < 5 miles.

4. Describe the graph of the function as either increasing ~~or decreasing~~.

Neither (constant)

You have just graphed a *step function*. A **step function** is a piecewise function whose pieces are disjoint constant functions.

5. Why do you think this function is called a step function?

Problem 3 Special Step Functions

The *greatest integer function* is a special kind of step function. The **greatest integer function**, also known as the **floor function**, $G(x) = \lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

1. Evaluate each using the greatest integer function.

a. $\lfloor 2 \rfloor = 2$

b. $\lfloor 0.17 \rfloor = 0$

c. $\lfloor 2.34 \rfloor = 2$

d. $\lfloor -1.2 \rfloor = -2$

e. $\lfloor 2.99999 \rfloor = 2$

f. $\lfloor -0.2 \rfloor = -1$

The *least integer function* is another special kind of step function. The **least integer function** $L(x) = \lceil x \rceil$ also known as the **ceiling function,** is defined as the least integer greater than or equal to x.

3. Calculate each:

a. $\lceil 2 \rceil = 2$

b. $\lceil 0.17 \rceil = 1$

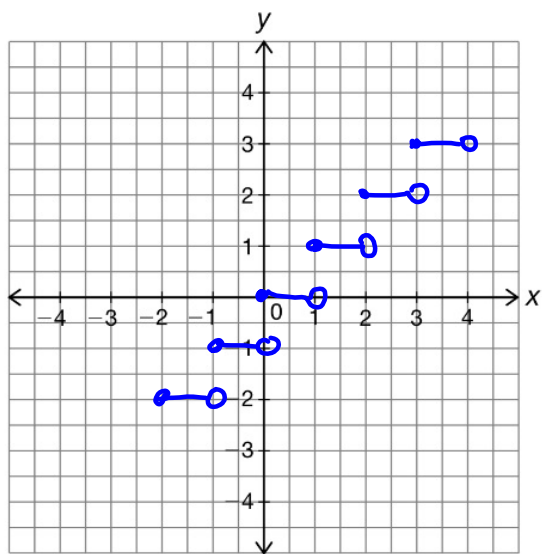**c.** $\lceil 2.34 \rceil = 3$

d. $\lceil -1.2 \rceil = -1$

e. $\lceil 2.99999 \rceil = 3$

f. $\lceil -0.2 \rceil = 0$

Graph $G(x) = \lfloor x \rfloor$.

Graph $L(x) = \lceil x \rceil$.