(10) $2x^4 + 7x^3 + 5x^2$

$x^2(\underbrace{2x^2 + 7x + 5})$

$x^2(2x + 5)(x + 1)$

$\underbrace{2x + 5x}_{7x}$

(15)

$(8x^4 + 8x^3) + (27x + 27)$

$8x^3(x + 1) + 27(x + 1)$

$(x+1)(\underbrace{8x^3 + 27}_{a^3 + b^3})$

$a = 2x \qquad b = 3$

$(x+1)(2x+3)(4x^2 - 6x + 9)$

$a^3 + b^3 =$
$\boxed{(a+b)(a^2 - ab + b^2)}$

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$$(4x^4 - 4x^3)(-x+1)$$

$$4x^3(x-1) - 1(x-1)$$

$$\boxed{(x-1)(4x^3-1)}$$

2-4 Dividing Polynomials

Objectives:

- I can divide polynomials using long division.

-I can divide polynomials using synthetic division.

Vocab

Divisor

$$\begin{array}{r} 23 \quad R{:}1 \\ 12\,\overline{)\,277} \leftarrow \text{Dividend} \\ -24\downarrow \\ \hline 37 \\ -36 \\ \hline 1 \end{array}$$

$$12 \cdot 23 + 1 = 277$$

Divisor

Quotient

R

Dividend

Dividing Polynomials - Long Division

Steps: 1. Write as a division problem w/ dividends & divisor in descending order, leaving spaces for missing terms in the dividend (0x)

2. Divide leading terms and write the result above the 1st term in the dividend

$$\begin{array}{r} x \\ \overline{)\quad} \end{array}$$

3. Multiply the result from #2 by the divisor & write the product under the dividend

4. Put ( ) around result from #3, distribute the subtraction sign & then add

5. Bring down remaining terms & repeat until there are no remaining terms in the dividend

6. Answer can be written in several ways

(B) $(6x^4 + 5x^3 + 2x + 8) \div (x^2 + 2x - 5)$

Write the dividend in standard form, including terms with a coefficient of 0.

$6x^4 + 5x^3 + 0x^2 + 2x + 8$

Write the division in the same way as you would when dividing numbers.

$6x^2 - 7x + 44$ R: $-121x + 228$

$(x^2 + 2x - 5)\overline{)6x^4 + 5x^3 + 0x^2 + 2x + 8}$

Divide.

$-6x^4 - 12x^3 + 30x^2$

$-7x^3 + 30x^2 + 2x$

$+7x^3 + 14x^2 - 35x$

$44x^2 - 33x + 8$

$-44x^2 - 88x + 220$

$-121x + 228$

Book Example 1A

$$\left(4x^3 + 2x^2 + 3x + 5\right) \div \left(x^2 + 3x + 1\right)$$

Your Turn

Use long division to find the quotient and remainder. Write the result in the form *dividend* = (*divisor*)(*qoutient*) + *remainder* and then carry out a check.

3. $(15x^3 + 8x - 12) \div (3x^2 + 6x + 1)$

Example: Divide the polynomial using long division.

$$\left(x^4 - 3x^3 + 6x^2 - 3x + 5\right) \div \left(x^2 + 1\right)$$

x^2

$(x^2 + 0x + 1) \overline{)\, x^4 - 3x^3 + 6x^2 - 3x + 5}$

$-\; x^4 - 0x^3 + x^2$

$-3x^3 + 5x^2 - 3x$

$x^2(0x) = 0x^3$

Dividing Polynomials - Synthetic division:

Can only be used to divide by a linear function

steps:

1. Write the terms of the dividend in descending order. Write the coeff. of the dividend in the first row using zeros for any missing terms not found in the dividend.
2. Write the zero, r, of the divisor (x-r),in the box.
3. Drop the 1st coeff. to the last row.
4. Multiply 1st coeff. by r & put product under the 2nd coeff.
5. Add product from #4 to 2nd coeff. & write the sum in the last row.
6. Repeat #4 & #5 until all coeff. have been used.
7. Write answer by putting variables behind the #'s in the last row. Start with 1 degree less than the dividend polynomial.

Ⓑ $(4x^4 - 3x^2 + 7x + 2) \div \left(x - \frac{1}{2}\right) = 4x^3 + 2x^2 - 2x + 6 \quad R: 5$

Find a. Then write the coefficients and a in the synthetic division format.

$4x^4 \;\; - 3x^2 + 7x + 2$

$+0x^3$

$+\frac{1}{2}\big| \quad 4x^4 \quad 0x^3 \quad -3x^2 \quad 7x \quad 2$

$+ \quad \downarrow \quad 2 \quad 1 \quad -1 \quad 3$

$4x^3 + 2x^2 - 2x + 6 \quad \boxed{5} \; R$

A $(7x^3 - 6x + 9) \div (x + 5)$

$$
\begin{array}{r|cccc}
-5 & 7x^3 & 0 & -6 & 9 \\
+ & \downarrow & -35 & 175 & -845 \\
\hline
 & 7x^2 & -35x & +169 & \boxed{-836} \\
 & & & & R
\end{array}
$$

Your Turn

Given a polynomial $p(x)$, use synthetic division to divide by $x - a$ and obtain the quotient and the (nonzero) remainder. Write the result in the form $p(x) = (x - a)(quotient) + p(a)$. You may wish to perform a check.

6. $(2x^3 + 5x^2 - x + 7) \div (x - 2)$

Example: Divide the polynomial using synthetic division.

$$\left(x^3+3x^2-4x-12\right)\div\left(x+3\right)$$

Long Division	Synthetic Substitution
$\begin{array}{r} 3x^2+10x+20 \\ x-2\,\overline{)\,3x^3+4x^2+0x+10} \\ \underline{-\left(3x^3-6x^2\right)} \\ 10x^2+0x \\ \underline{-\left(10x^2-20x\right)} \\ 20x+10 \\ \underline{-20x-40} \\ 50 \end{array}$	$\begin{array}{r\|rrrr} 2 & 3 & 4 & 0 & 10 \\ & & 6 & 20 & 40 \\ \hline & 3 & 10 & 20 & \boxed{50} \end{array}$

Example: Divide the polynomial using **any** method.

$$\left(x^3+4x^2+x-6\right)\div\left(x-1\right)$$

Verify the following polynomial identity

$$(a+b)(a^2-ab+b^2)=a^3+b^3$$

$$(a+b)(a^2-ab+b^2)=a^3+b^3$$

$$a(a^2)+a\left(\boxed{}\right)+a(b^2)+b(a^2)+\boxed{}(-ab)+b(b^2)=a^3+b^3$$

$$a^3-a^2b+ab^2+\boxed{}-ab^2+\boxed{}=a^3+b^3$$

$$a^3-\boxed{}+a^2b^2-\boxed{}+b^3=a^3+b^3$$

$$a^3\boxed{}b^3=a^3+b^3$$

Therefore, $(a+b)(a^2-ab+b^2)=a^3+b^3$ is a ______ statement.