3.1 Zeros of a Polynomial

Objectives:

- I can find the zeroes of a polynomial by using the factor theorem, remainder theorem, and rational roots theorem

Divide the following polynomials

$$x+4\overline{)3x^2+7x-20}$$

$3x-5$

$-3x^2 - 12x$

$-5x - 20$

$+5x + 20$

0

$\boxed{3x-5}$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

3 | 2 -5 7 -3 1

+ ↓ 6 3 30 81

$2x^3 + 1x^2 + 10x + 27$ | 82

$\boxed{2x^3 + x^2 + 10x + 27 \quad R: 82}$

Identify the zeros of the following and explain what that means graphically. what values of x make the factors equal 0?

$$f(x) = (x+2)(x-1)(x+3)$$

-2 1 -3

Zeros: $-2, 1, -3$ $x = -2, 1, -3$

Write the function in standard form and state the relationship between the degree and zeros of the function

$(x+2)(x-1)(x+3)$

$(x^2 - x + 2x - 2)$

$(x^2 + x - 2)(x+3) = x^3 + 3x^2 + x^2 + 3x - 2x - 6$

$= x^3 + 4x^2 + x - 6$

Degree $= 3$

# of Zeros $= 3$

Remainder Theorem:

For a polynomial $p(x)$ and a number a the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

Determine whether the given binomial is a factor of the polynomial p(x). If so, find the remaining factors of p(x).

(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5;\ (x+1)$

$$\begin{array}{r|rrrrr} -1 & 1x^4 & -4 & -6 & 4 & 5 \\ + & \downarrow & -1 & 5 & 1 & -5 \\ \hline & 1x^3 & -5x^2 & -1x & +5 & \boxed{0} \end{array}$$

R = factor!

$\boxed{(x+1) \text{ is a factor}}$

$(x^3 - 5x^2) + (-x + 5)$

$x^2(x-5) - 1(x-5)$

$(x-5)(x^2-1)$

$\boxed{(x-5)(x-1)(x+1)}$

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Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) $p(x) = x^3 + 3x^2 - 4x - 12;\ (x+3)$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ + & \downarrow & -3 & 0 & 12 \\ \hline & 1x^2 & +0x & -4 & \boxed{0} \end{array}$$

① $(x+3)$ is a factor

② $x^2 - 4 = \boxed{(x-2)(x+2)}$

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Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

x+4 is a factor!

8. $p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$

$$\begin{array}{r|rrrrr} -4 & 2 & 8 & 0 & 2 & 8 \\ + & \downarrow & -8 & 0 & 0 & -8 \\ \hline & 2x^3 & +0x^2 & +0x & +2 & 0 \end{array}$$

$2x^3+2$

$2(\underbrace{x^3+1})$

a^3+b^3

$a=x \quad b=1$

$\boxed{2(x+1)(x^2-x+1)}$

9. $p(x) = 3x^3 \overset{0x^2}{\wedge} - 2x + 5; (x - 1)$

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

no decimals

can be a fraction

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

$f(x) = 2x^3 - 7x^2 + 5x + 6$

Constant: 6 — Factors: 1, 2, 3, 6

Leading Coefficient: 2 — Factors: 1, 2

Possible Rational Roots:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

$\pm \frac{2}{2} \quad \pm \frac{6}{2}$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$f(x) = x^3 + 2x^2 - 19x - 20$

Possible R.R.

Costant: -20 → 1, 2, 4, 5, 10, 20

±1, ±2, ±4, ±5, ±10, ±20

L.C.: 1 → 1

Test (x-1)

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -19 & -20 \\ + & \downarrow & 1 & 3 & -16 \\ \hline & 1 & 3 & -16 & \boxed{-36} \end{array} \neq 0$$

Test (x+1)

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -19 & -20 \\ + & \downarrow & -1 & -1 & 20 \\ \hline & 1x^2 + & 1x & -20 & \boxed{0} \end{array} = 0$$

So (x+1) is a factor AND

-1 is a root/zero

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

Possible RR

±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

Constant: 24

Factors: 1, 2, 3, 4, 6, 8, 12, 24

L.C.: 1

Factors: 1

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & -7 & 22 & 24 \\ + & \downarrow & 4 & 0 & -28 & -24 \\ \hline & 1x^3 + & 0x^2 & -7x & -6 & \boxed{0} \end{array}$$

4 is a zero (root)

±1, 2, 3, 6

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -7 & -6 \\ + & \downarrow & -2 & 4 & 6 \\ \hline & 1x^2 & -2x & -3 & \boxed{0} \end{array}$$

-2 is a root

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

GCF: x

$f(x) = x(x^2 - 2x - 8)$

$(x)(x+2)(x-4)$

zeros: $\boxed{0, -2, 4}$

$x = 0$

$x + 2 = 0$ $x = -2$

$x - 4 = 0$ $x = 4$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

Possible R.R.

$\pm 1, \pm 2, \pm 4, \pm 8$

$\pm 1/2, \pm 2/2, \pm 4/2, \pm 8/2$

1 2 4

Constant: 8

1, 2, 4, 8

L.C.: 2

1, 2

$$\begin{array}{r|rrrrr} 4 & 2x^4 & -7 & -8 & 14 & 8 \\ + & \downarrow & 8 & 4 & -16 & -8 \\ \hline & 2x^3 & +1x^2 & -4x & -2 & \boxed{0} \end{array}$$

R

Factor by grouping

$(2x^3 + x^2) + (-4x - 2)$

$x^2(2x+1) - 2(2x+1)$

$(2x+1)(x^2-2)$

zeros: $-1/2, \pm\sqrt{2}, 4$

Set factors = 0 & solve

$2x + 1 = 0$
$-1 \quad -1$
$\frac{2x}{2} = \frac{-1}{2}$
$x = \frac{-1}{2}$

$x^2 - 2 = 0$
$+2 \quad +2$
$\sqrt{x^2} = \sqrt{2}$
$x = \pm\sqrt{2}$

① $\overset{L.C}{1}x^3 - x^2 - 10x - \overset{const.}{8}$

$1, 2, 4, 8$

Possible RR

$\pm 1, \pm 2, \pm 4, \pm 8$

Then graph!

zeros: $4, -1, -2$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -10 & -8 \\ & \downarrow & 4 & 12 & 8 \\ \hline & 1 & 3 & 2 & \boxed{0} \end{array}$$

$1x^2 + 3x + 2$

$(x+1)(x+2)$

$-1 \quad -2$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1