3.1 Zeros of a Polynomial

Objectives:

- I can find the zeroes of a polynomial by using the factor theorem, remainder theorem, and rational roots theorem

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Divide the following polynomials

$$
\begin{array}{r}
3x-5 \\
(x+4)\overline{)3x^2+7x-20} \\
-3x^2-12x \\
\hline
-5x-20 \\
+5x+20 \\
\hline
R:0
\end{array}
$$

$\boxed{3x-5}$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

$$
\begin{array}{r|rrrrr}
3 & 2x^4 & -5 & 7 & -3 & 1 \\
+ & \downarrow & 6 & 3 & 30 & 81 \\
\hline
 & 2x^3 & +1x^2 & +10x & +27 & 82
\end{array}
$$

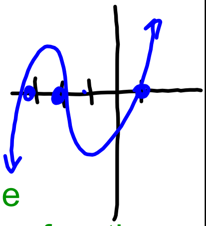
$$27+\frac{82}{x-3}$$

Identify the zeros of the following and explain what that means graphically. what value(s) of x makes $f(x) = 0$ (make each factor $=0$)

factored form $\rightarrow f(x) = (x+2)(x-1)(x+3)$

zeros: $x = -2, \ 1, \ -3$

The points where we cross x-axis



Write the function in standard form and state the relationship between the degree and zeros of the function

$\boxed{(x+2)(x-1)}(x+3)$

$(x^2 - x + 2x - 2)$

$(x^2 + x - 2)(x+3)$

$$\begin{array}{r} x^3 + 3x^2 \\ x^2 + 3x \\ -2x - 6 \\ \hline x^3 + 4x^2 + x - 6 \end{array}$$

Standard Form $\rightarrow \boxed{x^3 + 4x^2 + x - 6}$

Degree: 3
\# of zeros: 3
} Always match!

Remainder Theorem:

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

If remainder $=0$, then a is a zero

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

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Determine whether the given binomial is a factor of the polynomial p(x). If so, find the remaining factors of p(x).

(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5$; $(x + 1)$

Divide & See if it has R=0

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -6 & 4 & 5 \\ + & \downarrow & -1 & 5 & 1 & -5 \\ \hline & 1x^3 & -5x^2 & -1x & +5 & \boxed{0} \end{array}$$

Yes! (x+1) is a

$(x^3 - 5x^2) + (-x + 5)$

$x^2(x-5) - 1(x-5)$

$(x-5)(x^2-1) = \boxed{(x-5)(x+1)(x-1)}$

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Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) $p(x) = x^3 + 3x^2 - 4x - 12$; $(x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ + & \downarrow & -3 & 0 & 12 \\ \hline & 1x^2 & +0x & -4 & \boxed{0} \end{array}$$

$\boxed{(x+3) \text{ is a factor!}}$

$x^2 - 4 = \boxed{(x-2)(x+2)}$

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Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

8. $p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$

9. $p(x) = 3x^3 - 2x + 5; (x - 1)$

no variables

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

no decimals

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

$$p(x) = 2x^3 + 3x^2 - 7x + 6$$

Constant: 6 ← 1, 2, 3, 6

Leading coefficient: 2 ← 1, 2

Possible roots

$\pm 1, \pm 2, \pm 3, \pm 6$

$\pm 1/2, \pm 3/2$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^3 + 2x^2 - 19x - 20$$

Possible Roots (zeros):

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

const: -20

L.C.: 1

1, 2, 4, 5, 10, 20

1

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1