

Zeros of a Polynomial

Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

12. $p(x) = x^3 + 2x^2 - x - 2, (x+2)$

13. $p(x) = 2x^4 + 6x^3 - 5x - 10, (x+2)$

$$\begin{array}{r} +0x^2 \\ -2 \downarrow 2 \quad 6 \quad 0 \quad -5 \quad -10 \\ + \downarrow -4 \quad -4 \quad 8 \quad -6 \\ \hline 2 \quad 2 \quad -4 \quad 3 \quad -16 \end{array} \leftarrow \text{Remainder} \neq 0$$

Not a Factor

14. $p(x) = x^3 - 22x^2 + 157x - 360, (x-8)$ 15. $p(x) = 4x^3 - 12x^2 + 2x - 5, (x-3)$

8]
$$\begin{array}{r} 1 \quad -22 \quad 157 \quad -360 \\ + \downarrow \quad 8 \quad -112 \quad 360 \\ \hline 1 \quad -14 \quad 45 \quad 0 \end{array} \leftarrow \text{Remainder} = 0$$

x-8 is A factor

$x^2 - 14x + 45 = (x-5)(x-9)$

↑ other factors

$-5 \cdot -9 = 45$
 $-5 + -9 = -14$

7.1 Find all the zeros of the following polynomials

1. $f(x) = x^3 - x^2 - 10x - 8$ Possible zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

2. $f(x) = 2x^3 - x^2 - 13x - 6$ Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2$

$$\begin{array}{r} 3 \downarrow 2 \quad -1 \quad -13 \quad -6 \\ + \downarrow \quad 6 \quad 15 \quad 6 \\ \hline 2x^2 + 5x + 2 \quad 0 \end{array}$$

$(2x+1)(x+2)$

↑ ↑
-1/2 -2

zeros: $3, -1/2, -2$

3. $g(x) = x^3 - 9x^2 + 23x - 15$ Possible zeros: $\pm 1, \pm 3, \pm 5, \pm 15$

1]
$$\begin{array}{r} 1 \quad -9 \quad 23 \quad -15 \\ + \downarrow \quad 1 \quad -8 \quad 15 \\ \hline 1 \quad -8 \quad 15 \quad 0 \end{array}$$

$x^2 - 8x + 15 = (x-5)(x-3)$

↑ ↑
-5 -3

zeros: 1, 3, 5

4. $h(x) = 6x^3 - 7x^2 - 9x - 2$

5. $g(x) = x^4 - 6x^3 + 11x^2 - 6x$

6. $g(x) = x^4 - 5x^2 + 4$ Possible zeros: $\pm 1, \pm 2, \pm 4$

1]
$$\begin{array}{r} 1 \quad 0 \quad -5 \quad 0 \quad 4 \\ \downarrow 1 \quad 1 \quad -4 \quad -4 \\ \hline 1 \quad 1 \quad -4 \quad -4 \quad 0 \\ \downarrow -1 \quad 0 \quad 4 \\ \hline 1 \quad 0 \quad -4 \quad 0 \end{array}$$

$x^2 - 4 = (x+2)(x-2)$

↑ ↑
-2 2

↘ $(x^3+x^2)+4x-4$
 $x^2(x+1)-4(x+1)$
 $(x^2-4)(x+1)$

zeros: 1, -1, -2, 2

7. $f(x) = x^3 - 4x^2 - 11x + 2$ Possible zeros: $\pm 1, \pm 2$ 8. $f(x) = x^3 - 4x^2 + 2x + 4$

-2	1	-4	-11	2	Zeros: -2, $3 \pm 2\sqrt{2}$
↓	-2	12	-2		
	1	-6	1	0	

Irrational

$x^2 - 6x + 1$ Not factorable!

Quadratic formula: $\frac{6 \pm \sqrt{36 - 4}}{2} = \frac{6 \pm \sqrt{32}}{2}$ $\frac{8\sqrt{2}}{4}$

17. Identify the zeroes of $f(x) = (x+3)(x-4)(x-3)$, write the function in standard form, and state how the zeros are related to the standard form.

$f(x) = (x+3)(x-4)(x-3) = (x^2 - x - 12)(x-3) = x^3 - 3x^2 - x^2 + 3x - 12x + 36$

Zeros: -3, 4, 3	Standard Form: $x^3 - 4x^2 - 9x + 36$
-----------------	---------------------------------------

The # of zeroes is the same as the degree

19. Explain the Error Sabrina was told to find the zeros of the polynomial function

$h(x) = x(x-4)(x+2)$. She stated that the zeros of this polynomial are $x = 0$, $x = -4$, and $x = 2$. Explain her error.

~~$h(x) = x(x-4)(x+2)$~~

Review

Look @ Unit 2!

Perform the following operations

1. $(x^2 - 3x + 7) + (3x^2 + 5x - 3)$

2. $(-3x^2 - 5) - (x^2 + 7x + 12)$

3. $(5x^3 - 1)^2$ $(5x^3 - 1)(5x^3 - 1)$