(5) $g(x) = x^4 - 6x^3 + 11x^2 - 6x$

$x(1x^3 - 6x^2 + 11x - 6)$

0 L.C. const

Possible RR

$\pm 1, \pm 2, \pm 3, \pm 6$

3	1	-6	11	-6
+	↓	3	-9	6
	$1x^2$	$-3x$	$+2$	0

$x^2 - 3x + 2$

$(x-1)(x-2)$

1 2

$\frac{3 \pm \sqrt{9-8}}{2}$

$\frac{3 \pm \sqrt{1}}{2}$ $\frac{3+1}{2}$ $\frac{3-1}{2}$

zeros: 0, 3, 1, 2

(6) $x^4 - 5x^2 + 4$

LC const

Possible RR

$\pm 1, \pm 2, \pm 4$

2	1	0	-5	0	4
+	↓	2	4	-2	-4
	$1x^3$	$+2x^2$	$-1x$	-2	0

$(x^3 + 2x^2)(-x - 2)$

$x^2(x+2) - 1(x+2)$

$(x+2)(x^2-1)$

$(x+2)(x+1)(x-1)$

-2 -1 +1

3-2 Graphing Polynomial Functions

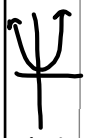
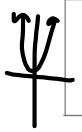
(Book 5.4 pg. 293-306)

Objectives:

- I can graph a polynomial function by hand and using technology
- I can find end behavior of a polynomial function
- I can identify zeros, x-intercepts, and factors of a polynomial function
- I can determine the multiplicity of a polynomial function

End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

	Function	Domain	Range	End Behavior
	$f(x) = x^{0}$	$(-\infty, \infty)$	$[0, \infty)$	R.E. As $x \to +\infty$, $f(x) \to \infty$. ↑ L.E. As $x \to -\infty$, $f(x) \to \infty$. ↑
	$f(x) = x^{0}$	$(-\infty, \infty)$	$[0, \infty)$	As $x \to +\infty$, $f(x) \to \infty$. ↑ As $x \to -\infty$, $f(x) \to \infty$. ↑
	$f(x) = x^{0}$	$(-\infty, \infty)$	$[0, \infty)$	As $x \to +\infty$, $f(x) \to \infty$. ↑ As $x \to -\infty$, $f(x) \to \infty$. ↑

Even exponents

Does it change if I have a negative coefficient? How?

Negative flips both ends down

Range: $(-\infty, 0]$

End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

Function	Domain	Range	End Behavior
$f(x) = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \to +\infty$, $f(x) \to \infty$. ↑ As $x \to -\infty$, $f(x) \to -\infty$ ↓
$f(x) = x^3$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \to +\infty$, $f(x) \to \infty$. ↑ As $x \to -\infty$, $f(x) \to -\infty$ ↓
$f(x) = x^5$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \to +\infty$, $f(x) \to \infty$. ↑ As $x \to -\infty$, $f(x) \to -\infty$. ↓

Odd exponents = ends opposite

Does it change if I have a negative coefficient? How?

Flips it upside down

End Behavior Game!!!

$4x^{11}$

L.C. matches the Right End
(+/−)

Even degree = ends match
Odd degree = ends opposite

Zeros, x-intercepts, and factors

Find the factors of $f(x) = \cancel{x^2 + 4x + 3}$

$(x+3)(x+1)$

Now find the x-intercepts of $f(x) = x^2 + 4x + 3$

point

$(-3, 0)$ $(-1, 0)$

Lastly find the zeros of $f(x) = x^2 + 4x + 3$

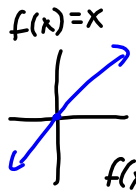
$x = -3, -1$

What is the same between the factors, x-intercepts, and zeros of this function?

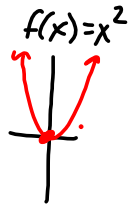
Multiplicity = Number

The **power** of the factor determines the nature of the intersection at the point $x = a$.
(This is referred to as the multiplicity.)

Straight intersection:
$(x - a)^1$ The power of the zero is 1.

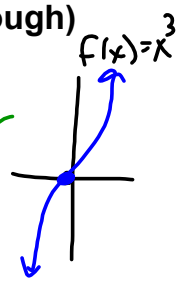


Tangent intersection :(bounce)
$(x - a)^{\text{even}}$ The power of the zero is even.



Inflection intersection: **(like a slide through)**
$(x - a)^{\text{odd}}$ The power of the zero is odd.

and greater than 1



(A) Use a graphing calculator to graph the cubic functions $f(x) = x^3$, $f(x) = x^2(x-2)$, and $f(x) = x(x-2)(x+2)$. Then use the graph of each function to answer the questions in the table.

Function	$f(x) = x^3$	$f(x) = x^2(x-2)$	$f(x) = x(x-2)(x+2)$
How many distinct factors does $f(x)$ have?			
What are the graph's x-intercepts?			
Is the graph tangent to the x-axis or does it cross the x-axis at each x-intercept?			
How many turning points does the graph have?			
How many global maximum values? How many local?			
How many global minimum values? How many local?			

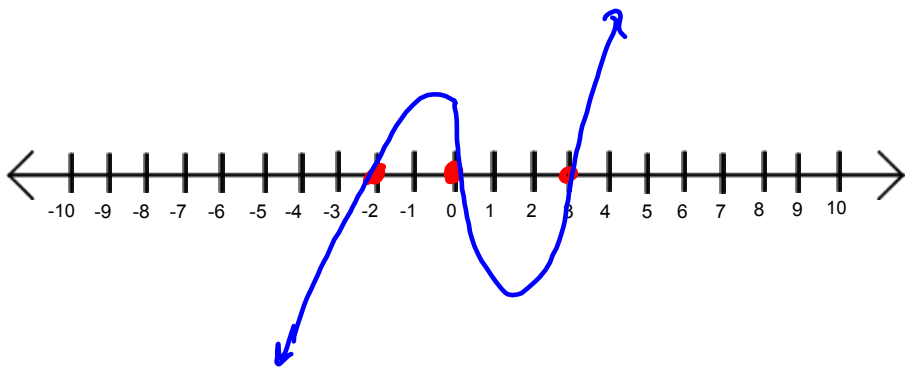
Sign Chart

Degree: 3

E.B.: ↓↑

$$f(x) = (x)(x+2)(x-3)$$

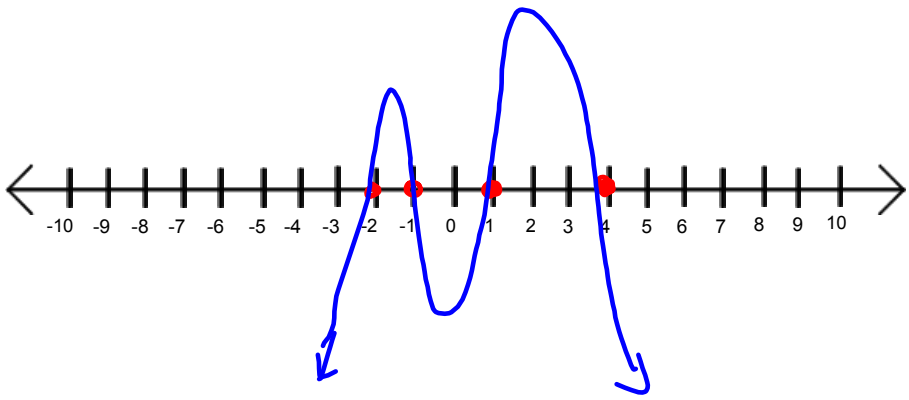
zeros: $x = 0, -2, +3$



Sign Chart Degree: 4 L.C. − E.B. ↓↓

$$f(x) = -(x-4)(x-1)(x+1)(x+2)$$

zeros: x= 4 , 1 , −1, −2



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Graphing a Polynomial from factors

(B) $f(x) = -(x-4)(x-1)(x+1)(x+2)$

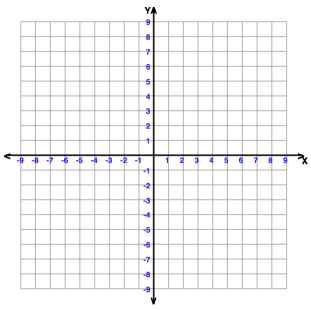
Identify the end behavior.

As $x \to +\infty$, $f(x) \to$ ____.

As $x \to -\infty$, $f(x) \to$ ____.

Identify the graph's x-intercepts, and then use the sign of $f(x)$ on intervals determined by the x-intercepts to find where the graph is above the x-axis and where it's below the x-axis.

The x-intercepts are $x =$ ____, $x =$ ____, $x =$ ____, $x =$ ____.



Ex. 8 Find the zeros, the multiplicity, end behavior and graph the following:

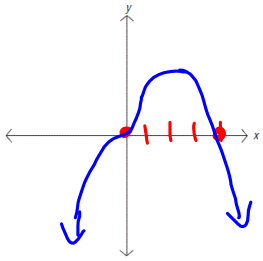
a. $f(x) = -x^3(x-4)$

$-x^3(x-4) = -x \cdot x \cdot x(x-4)$

zeros: 0, 4

multip.: 3 + 1

E.B: ↓↓ Degree = 4



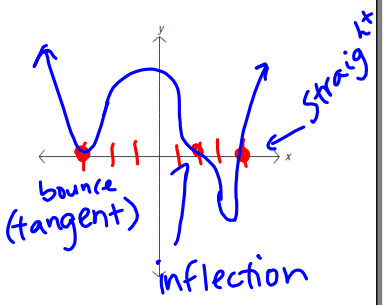
b. $f(x) = (x+3)^2(x-2)^3(x-4)$

$(x+3)^2(x-2)^3(x-4)$

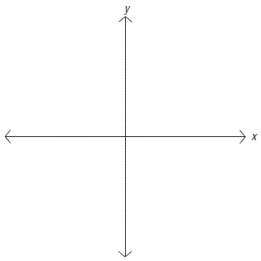
zeros: −3, 2, 4

multiplicity: 2, 3, 1

Degree: 6 E.B.: ↑↑



c. $f(x) = (x+2)^3(x-1)^2$



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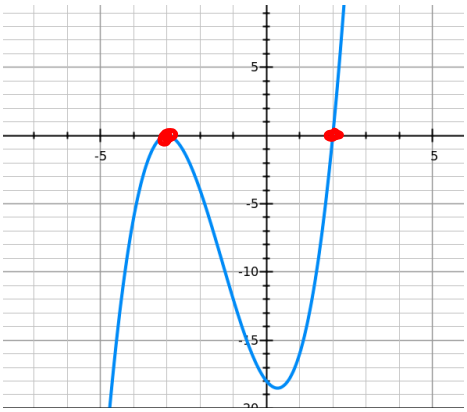
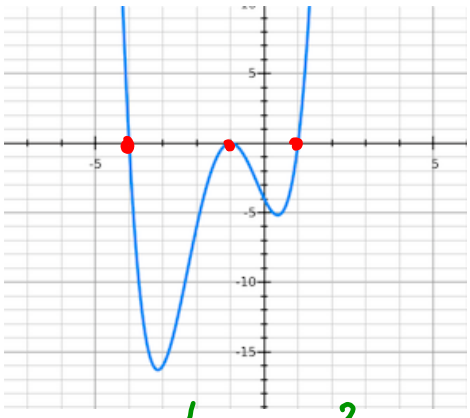
Write a function in intercept form for the given graphs whose intercepts are integers. Assume the constant factor of a is either 1 or -1.



−3 2

$f(x) = (x+3)^2(x-2)^1$



$f(x) = (x+4)^1(x+1)^2(x-1)^1$

5. $f(x)=(x-5)(x-4)(x+1)$

$x = 5, 4, -1$

multiplicity: 1, 1, 1

multiplicity = exponent of factor

Degree = 3 (odd = opp) E.B.: L ↓ R ↑

End Behavior:

L.E. $\lim_{x\to-\infty} f(x) = -\infty$ ↓

$\lim_{x\to\infty} f(x) = \infty$ ↑

8. $f(x)=-x(x+2)^2(x-2)^3$

$x = 0, -2, 2$

1, 2, 3

$\underbrace{-x}_{1}\underbrace{(x+2)(x+2)}_{2}\underbrace{(x-2)(x-2)(x-2)}_{3}$

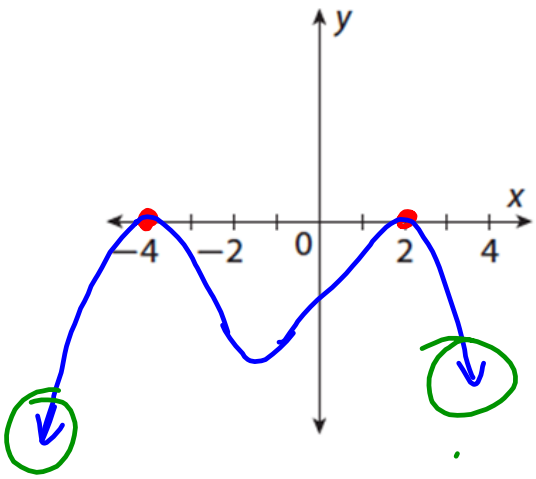
Degree = 6 L.C. = − E.B.: L ↓ R ↓

End Behavior:

$\lim_{x\to-\infty} f(x) = -\infty$ ↓

$\lim_{x\to\infty} f(x) = -\infty$ ↓

10. $f(x) = -(x-2)^2(x+4)^2$



zeros:

$x = 2, -4$

2 tangent

2 tangent

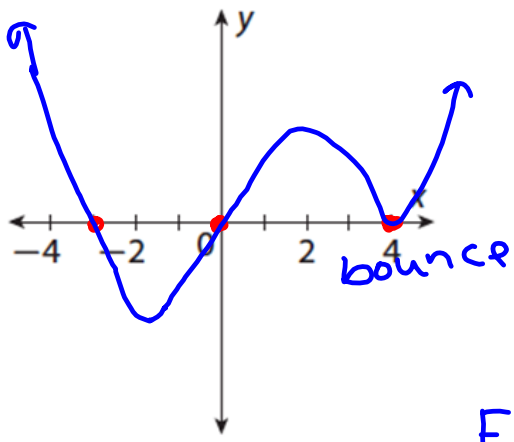
Deg: 4

L.C.: −

E B: ↓ ↓

11.

$f(x) = x(x-4)^2(x+3)$



$x = 0, 4, -3$

1 Straight

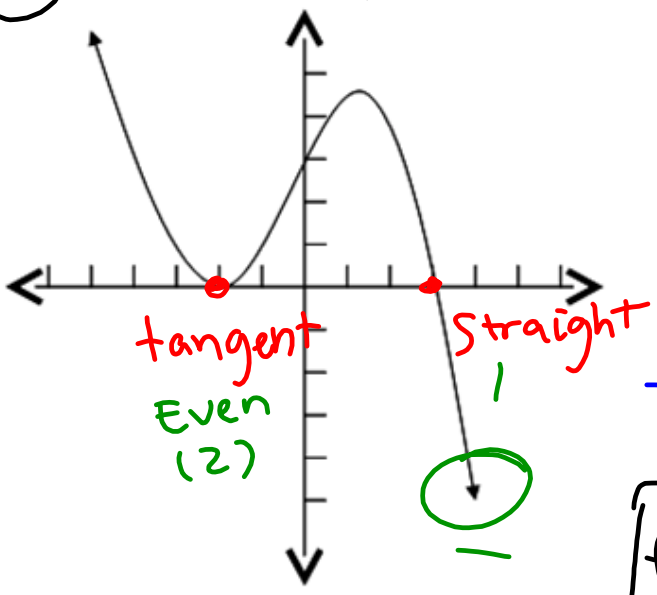
2 tangent (bounce)

1 straight

E.B. ↑ ↑

Deg = 4

(13) Write polynomial



$x = -2, 3$

$2 \quad 1$

$-(x+2)^2(x-3)^1$

$$\boxed{f(x) = -(x+2)^2(x-3)}$$