3-2 Graphing Polynomial Functions

(Book 5.4 pg. 293-306)

Objectives:

- I can graph a polynomial function by hand and using technology
- I can find end behavior of a polynomial function
- I can identify zeros, x-intercepts, and factors of a polynomial function
- I can determine the multiplicity of a polynomial function

End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

Function	Domain	Range	End Behavior
$f(x) = x^2$	$(-\infty, \infty)$	$[0, \infty)$	RE As $x \to +\infty$, $f(x) \to \boxed{\infty}$. ↑ As $x \to -\infty$, $f(x) \to \boxed{\infty}$. ↑ LE
$f(x) = x^4$	$(-\infty, \infty)$	$[0, \infty)$	As $x \to +\infty$, $f(x) \to \boxed{\infty}$. ↑ As $x \to -\infty$, $f(x) \to \boxed{\infty}$. ↑
$f(x) = x^6$	$(-\infty, \infty)$	$[0, \infty)$	As $x \to +\infty$, $f(x) \to \boxed{\infty}$. ↑ As $x \to -\infty$, $f(x) \to \boxed{\infty}$. ↑

Even Degree = Ends go same direction

Does it change if I have a negative coefficient? How?

Flip the ends upside down

End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

Function	Domain	Range	End Behavior
$f(x) = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \to +\infty$, $f(x) \to \boxed{\infty}$. ↑ As $x \to -\infty$, $f(x) \to \boxed{-\infty}$. ↓
$f(x) = x^3$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \to +\infty$, $f(x) \to \boxed{\infty}$. ↑ As $x \to -\infty$, $f(x) \to \boxed{-\infty}$. ↓
$f(x) = x^5$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \to +\infty$, $f(x) \to \boxed{\infty}$. ↑ As $x \to -\infty$, $f(x) \to \boxed{-\infty}$. ↓

Odd Degree = Ends go opposite directions

Does it change if I have a negative coefficient? How?

Flip the ends

↑ ↓

End Behavior Game!!!!!

$$-3x^4$$

Zeros, x-intercepts, and factors

Find the factors of $f(x) = x^2 + 4x + 3$

$(x+1)(x+3)$

Now find the x-intercepts of $f(x) = x^2 + 4x + 3$

(x, y)

$(-1, 0)$ $(-3, 0)$

Lastly find the zeros of $f(x) = x^2 + 4x + 3$

$x = -1, -3$

What is the same between the factors, x-intercepts, and zeros of this function?

Number Multiplicity

The **power** of the factor determines the nature of the intersection at the point $x = a$.
(This is referred to as the multiplicity.)

Straight intersection:

$(x - a)^1$ The power of the zero is 1.

Tangent intersection : (bounce)

$(x - a)^{even}$ The power of the zero is even.

Inflection intersection: **(like a slide through)**

$(x - a)^{odd}$ The power of the zero is odd.

$$f(x) = (x)(x+2)(x-3)$$

$x = 0, -2, 3$

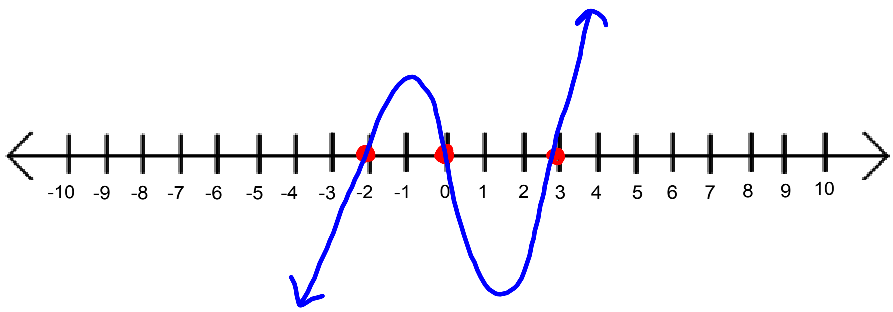
exponents → $\underbrace{m1 \quad m1 \quad m1}_{\text{Straight}}$

E.B. L ↓ R ↑

L.C +

Deg: 3

x-axis



L.C.

$$f(x) = -(x-4)(x-1)(x+1)(x+2)$$

$x = 4, 1, -1, -2$

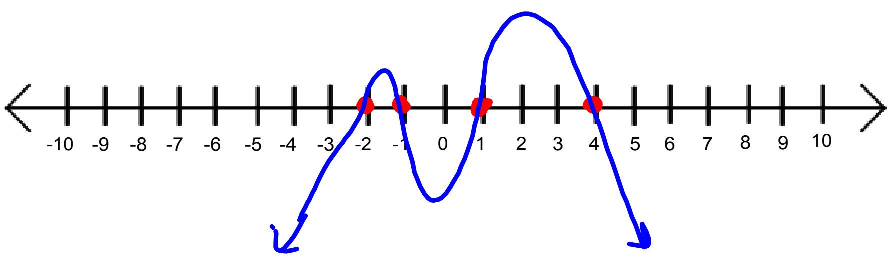
$\underbrace{m1 \quad m1 \quad m1 \quad m1}_{\text{Straight}}$

E.B: L ↓ R ↓

L.C: −

Deg: 4

R. End Match



Ex. 8 Find the zeros, the multiplicity, end behavior and graph the following:

exponents

a. $f(x) = -x^2(x-4)$

$f(x) = -x^2(x-4)$

$-1 \cdot x \cdot x \cdot (x-4)$

$\boxed{x = 0, 4}$

m 2 m 1

tangent straight

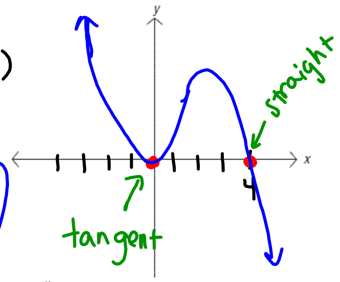
E.B. ↑ ↓

as $x \to -\infty$, $f(x) \to \infty$

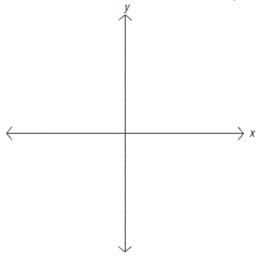
as $x \to \infty$, $f(x) \to -\infty$

L.C. −

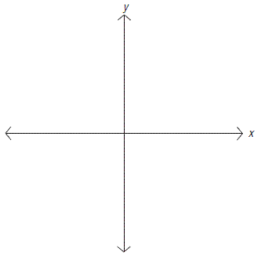
Deg: 3



b. $f(x) = (x+3)^2(x-2)^3(x-4)$



c. $f(x) = (x+2)^3(x-1)^2$



$f(x) = (x+3)^2(x-2)^3(x-4)$

$(x+3)^2(x-2)^3(x-4)$

$\boxed{x = -3, 2, 4}$

m 2 m3 m 1

tangent inflection straight

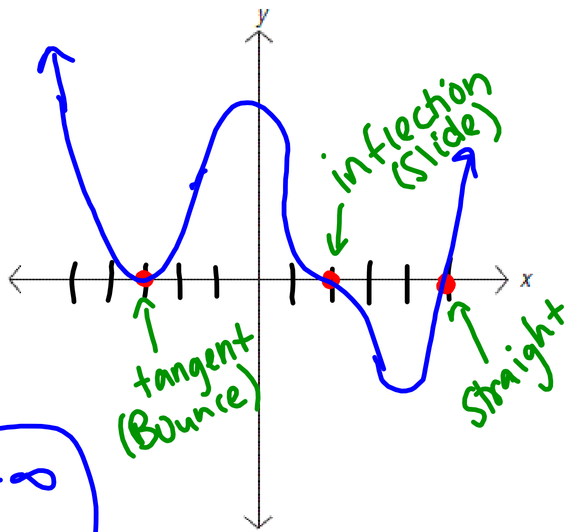
E.B. ↑↑

as $x \to -\infty$ $f(x) \to +\infty$

as $x \to \infty$ $f(x) \to \infty$

L.C. + R.E.

Deg: $2+3+1=6$ (match) (even)



$f(x) = (x+2)^3(x-1)^2$

$(x+2)^{3}(x-1)^{2}$

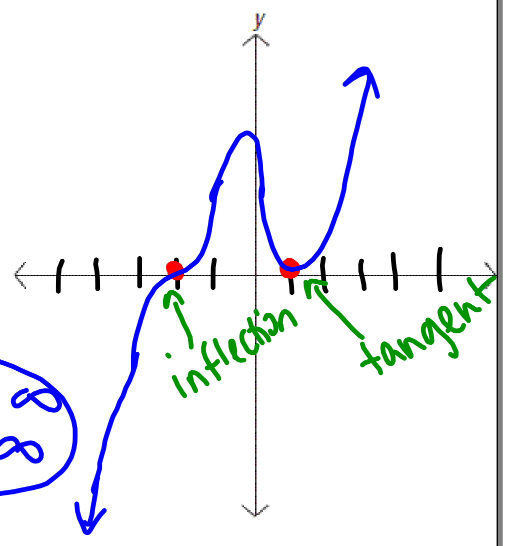
$x = -2, 1$

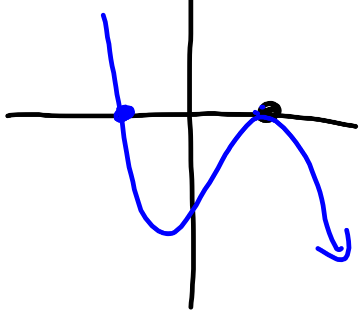
m 3 inflection m 2 tangent

E.B. $\downarrow \uparrow$ as $x \to -\infty$ $f(x) \to -\infty$
as $x \to \infty$ $f(x) \to \infty$

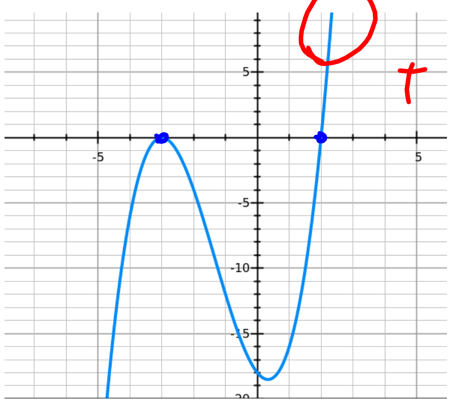
L.C: + (Right)

Deg: 5 (Left is Opp)





Write a function in ~~intercept~~ Factored form for the given graphs whose intercepts are integers. Assume the ~~constant factor~~ Leading coefficient of a is either 1 or -1.

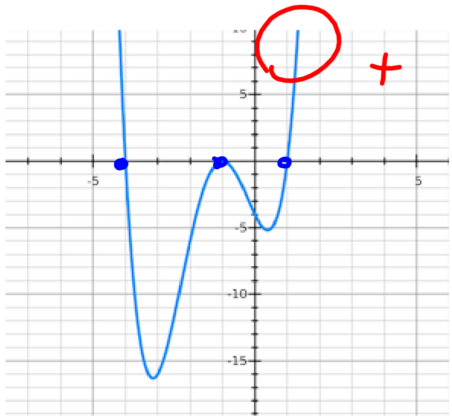


+

$x = -3, 2$

tangent m 2, straight m 1

$$\boxed{f(x) = +(x+3)^2(x-2)}$$



+

$x = -4, -1, 1$

straight m 1, tangent m 2, straight m 1

$$\boxed{f(x) = +(x+4)(x+1)^2(x-1)}$$