

4-2: Adding and Subtracting Rational Expressions (Book 9.1)

Objectives:

1. I can simplify a rational expression
2. I can add and subtract rational expressions.

Adding Rational Numbers:

$$\frac{2}{2} \cdot \frac{3}{5} + \frac{3}{10} =$$

$$\frac{6}{10} + \frac{3}{10} = \frac{9}{10}$$

$$\frac{-2}{3} + \frac{-5}{6}$$

$$-\frac{2}{3} \cdot \frac{2}{2} = -\frac{4}{6}$$

$$-\frac{4}{6} + \frac{-5}{6} = \frac{-9}{6}$$

$$\frac{-3}{2}$$

$$\frac{1}{4} + \frac{9}{7} =$$

$$\frac{7}{7} \cdot \frac{1}{4} = \frac{7}{28}$$

$$\frac{4}{4} \cdot \frac{9}{7} = \frac{36}{28}$$

$$\frac{7}{28} + \frac{36}{28} = \frac{43}{28}$$

$$\frac{4}{7} + \frac{2 \cdot 7}{7 \cdot 7}$$

$$\frac{4}{7} + \frac{14}{7} = \frac{18}{7}$$

Adding Rational Expressions:

$$\frac{1}{x} + \frac{2}{x} = \frac{3}{x}$$

$$\frac{2}{(x-3)} + \frac{5}{(x)}$$

$$\frac{2}{(x-3)} \cdot \frac{x}{x} = \frac{2x}{x(x-3)}$$

$$\frac{5}{x} \cdot \frac{(x-3)}{(x-3)} = \frac{5x-15}{x(x-3)}$$

$$\frac{2x}{x(x-3)} + \frac{5x-15}{x(x-3)} = \frac{7x-15}{x(x-3)}$$

$$\frac{x}{(x+1)} + \frac{5}{(x-2)}$$

$$\frac{(x-2)}{(x-2)} \cdot \frac{x}{(x+1)} = \frac{x^2-2x}{(x-2)(x+1)}$$

$$\frac{(x+1)}{(x+1)} \cdot \frac{5}{(x-2)} = \frac{5x+5}{(x+1)(x-2)}$$

$$\frac{x^2-2x}{(x-2)(x+1)} + \frac{5x+5}{(x-2)(x+1)} = \frac{x^2+3x+5}{(x-2)(x+1)}$$

Given two or more rational expressions, the least common denominator (LCD) is found by factoring each denominator and finding the least common multiple (LCM) of the factors. This technique is useful for the addition and subtraction of expressions with unlike denominators.

Least Common Denominator (LCD) of Rational Expressions

To find the LCD of rational expressions:

1. Factor each denominator completely. Write any repeated factors as powers.
2. List the different factors. If the denominators have common factors, use the highest power of each common factor.

Find the LCD for each rational expression

$$\frac{-14}{\cancel{(x-3)}(x-8)} + \frac{9}{\cancel{(x-3)}}$$

$$\text{LCD: } (x-3)(x-8)$$

$$\frac{-2}{\textcircled{3x-15}} + \frac{6x}{\textcircled{4x+28}}$$

$$3(x-5) \quad 4(x+7)$$

$$\text{LCD: } \textcircled{3}(x-5)\textcircled{\cdot 4}(x+7)$$

$$12(x-5)(x+7)$$

Add the following rational expressions.
Identify any excluded values.

$$x \neq 3$$

$$\frac{x+6}{\textcircled{8x-24}} + \frac{14x}{\textcircled{10x-30}}$$

$$\cancel{8(x-3)} \quad \textcircled{10}\cancel{(x-3)}$$

$$\text{LCD: } 8(x-3) \cdot 10$$

$$\textcircled{80(x-3)} = 8 \cdot \underline{10}(x-3)$$

$$\frac{10(x+6)}{10 \cdot 8(x-3)} + \frac{\frac{8}{8} \cdot 14x}{8 \cdot 10(x-3)}$$

$$\frac{10x+60}{10 \cdot 8 \cdot (x-3)} + \frac{112x}{8 \cdot 10 \cdot (x-3)} = \frac{122x+60}{80(x-3)}$$

Add the following rational expressions.
Identify any excluded values.

$$\frac{1}{x-3} + \frac{2}{(x-3)(x-1)}$$

$$x \neq 3, 1$$

$$LCD: (x-3)(x-1)$$

$$\frac{(x-1)}{(x-1)} \cdot \frac{1}{(x-3)} + \frac{2}{(x-3)(x-1)}$$

$$\frac{x-1}{(x-1)(x-3)} + \frac{2}{(x-3)(x-1)} = \frac{x+1}{(x-3)(x-1)}$$

Add the following rational expressions.
Identify any excluded values.

$$\frac{4}{x} + \frac{5x}{x+5}$$

$$x \neq 0, -5$$

$$LCD: (x)(x+5)$$

$$\frac{(x+5)}{(x+5)} \cdot \frac{4}{x} + \frac{x}{x} \cdot \frac{5x}{(x+5)}$$

$$\frac{4x+20}{x(x+5)} + \frac{5x^2}{x(x+5)} = \frac{5x^2+4x+20}{x(x+5)}$$

Add the following rational expressions.
Identify any excluded values.

$$\frac{(x+6) \cdot 5}{(x+6)(x+3)(x-1)} + \frac{4x}{(x+3)(x+6)} \cdot \frac{(x-1)}{(x-1)}$$

$$\text{LCD: } (x+3)(x-1)(x+6)$$

$$x \neq -3, 1, -6$$

$$\frac{5x+30}{(x+3)(x-1)(x+6)} + \frac{4x^2-4x}{(x+3)(x-1)(x+6)}$$

$$\frac{4x^2 + x + 30}{(x+3)(x-1)(x+6)}$$

Add the following rational expressions.
Identify any excluded values.

$$\frac{5}{(x+2)(x+1)} + \frac{(x+3)}{(x+2)}$$

Add the following rational expressions.
Identify any excluded values.

$$\frac{4}{(x+5)} + \frac{x}{(x+5)}$$

Subtracting Rationals

Subtract the rational numbers

$$\frac{3}{5} - \frac{3}{10}$$

$$\frac{5}{4} - \frac{3}{8}$$

$$-\frac{2}{3} - \frac{5}{9}$$

$$-\frac{2}{5} - 2$$

Subtract the Rational Expressions

$$\frac{1}{x-2} - \frac{2}{x}$$

$$\frac{x}{5} - \frac{3}{x+1}$$

$$\frac{(x+1)}{(x+2)} - \frac{4}{(x+1)}$$

Subtract the following rational expressions.
Identify any excluded values.

$$\frac{12x}{15x + 60} - \frac{5}{(x + 5)(x + 4)}$$

Subtract and identify any excluded values.

$$\frac{x^2}{x + 4} - \frac{1}{x + 2}$$

subtract the following rational expressions.
Identify any excluded values.

$$\frac{3x}{x+5} - \frac{7}{(x+5)(x-2)}$$

Subtract the following rational expressions.
Identify any excluded values.

$$\frac{3x}{(x-6)(x+1)} - \frac{2}{(x+1)(x+1)}$$