4.2 Complex Zeros

- I can find all zeros of a polynomial including non-real complex zeros
- I can write a polynomial from its zeros
- I can do a linear factorization

Fundamental Thm of Alg: an nth degree polynomial will have n complex zeros

both real & non-real

(May be a combination of real and non-real complex. Some zeros may be repeated)

Complex Conjugates: complex imaginary factors come in conjugate pairs

(if 3i is a zero, - 3i is also)

Odd functions will always have at least one real zero - why??

Find all zeros of $p(x) = x^3 - 125$. Include any multiplicities greater than 1.

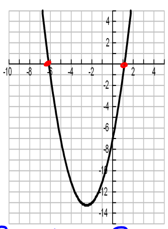
First factor the difference of two cubes.

Find all zeros of $p(x) = x^4 - 256$. Include multiplicities greater than 1.

Find use factoring patterns to factor the polynomial.

How many complex zeros does each function have? How many are real? How many are non-real?

$x^2 + 5x - 7$

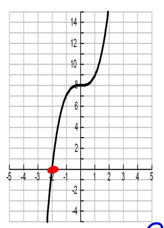


Complex: 2

Real: 2

Non-real: 0

$x^3 + 8$

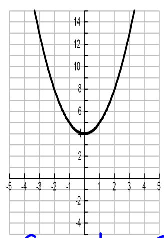


Complex: 3

Real: 1

Non-real: 2

$x^2 + 4$



Complex: 2

Real: 0

Non-real: 2

Linear Factorization Thm: a polynomial of nth degree has n linear factors

(some factors may be complex imaginary)

$$x^4 - 6x^3 + 10x^2 - 6x + 9$$

$$(x-3)(x+3)(x-i)(x+i)$$

4 factors

Factors:

(x - zero)

* Imaginary zeros come in conjugate pairs

Write a polynomial function of minimum degree with the following zeros and multiplicities:

4, 7, 2i, -2i

$(x-4)(x-7)(x-2i)(x+2i)$

$(x-(-2i))$

- 4, 2 + 3i, 2 - 3i

$(x+4)(x-(2+3i))(x-(2-3i))$

$(x-2-3i)(x-2+3i)$

3 with multi of 2

5 + i with multi of ~~1~~ 3

5 - i " " 3

$(x-3)^2(x-(5+i))^3(x-(5-i))^3$

Find all zeros and write a linear factorization of the following polynomial:

$$x^3 + 5x^2 + x + 5$$

$x = -5, i, -i$

zero

$$\begin{array}{r|rrrr} -5 & 1 & 5 & 1 & 5 \\ & \downarrow & -5 & 0 & -5 \\ \hline & 1x^2 & +0x & +1 & \boxed{0} \end{array}$$

$x = \frac{0 \pm \sqrt{0-4(1)(1)}}{2(1)}$

$\frac{\pm\sqrt{-4}}{2} = \frac{\pm 2i}{2} = \pm i$

$$x^3 - 11x^2 + 49x - 75$$

$x = 3, 4+3i, 4-3i$

zero $(x-3)$

$$\begin{array}{r|rrrr} 3 & 1x^3 & -11 & 49 & -75 \\ & \downarrow & 3 & -24 & 75 \\ \hline & 1x^2 & -8x & +25 & \boxed{0} \end{array}$$

$x = \frac{8 \pm \sqrt{64-4(1)(25)}}{2(1)} = \frac{8 \pm \sqrt{64-100}}{2} = \frac{8 \pm \sqrt{-36}}{2} = \frac{8 \pm 6i}{2}$

$\frac{8}{2} \pm \frac{6i}{2}$

$4 \pm 3i$

$(x-3)(x-(4+3i))$

$(x-(4-3i))$

$(x-3)(x-4-3i)(x-4+3i)$

Find all zeros and write a linear factorization of the following polynomial:

$$x^4 + x^3 + 5x^2 - x - 6$$

$x = -1, 1, \frac{-1+\sqrt{23}i}{2}, \frac{-1-\sqrt{23}i}{2}$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 5 & -1 & -6 \\ & \downarrow & 1 & 2 & 7 & 6 \\ \hline -1 & 1x^3 & +2x^2 & +7x & +6 & \boxed{0} \\ & + \downarrow & -1 & -1 & -6 & \\ \hline & 1x^2 & +1x & +6 & \boxed{0} & \end{array}$$

$x = \frac{-1 \pm \sqrt{1-4(1)(6)}}{2(1)} = \frac{-1 \pm \sqrt{1-24}}{2}$

$= \frac{-1 \pm \sqrt{-23}}{2} = \frac{-1 \pm \sqrt{23}i}{2}$

$(x+1)(x-1)\left(x-\left(\frac{-1+\sqrt{23}i}{2}\right)\right)\left(x-\left(\frac{-1-\sqrt{23}i}{2}\right)\right)$

Use the given zero to find the remaining zeros and write a linear factorization:

$2i$; $x^4 + 10x^3 + 38x^2 + 40x + 136$

$-2i$

$2i$	1	10	38	40	136
+	↓	$2i$	$-4+20i$	$-40+68i$	-136
$-2i$	1	$10+2i$	$34+20i$	$68i$	0
+	↓	$-2i$	$-20i$	$-68i$	
	1	10	34	0	

$1x^2 + 10x + 34$

$x^4 - 16$

1 0 0 0 −16