5-1 Rational Functions

Objectives:

- I can determine the domain, range, end behavior, and intervals of increasing and decreasing of rational functions.
- I can identify the transformation of a given function and sketch a graph
- I can write a rational equation given a graph.

State the domain of $f(x) = \frac{1}{x}$ x's $x \neq 0$

The function accepts all real numbers except 0, because division by 0 is undefined. So, the function's domain is as follows:

- As an inequality: $x <$ 0 or $x >$ 0
- In set notation: $\{x \mid x \neq 0\}$
- In interval notation (where the symbol $\cup$ means *union*):

$(-\infty, 0) \cup (0, +\infty)$

Domain: $(-\infty, 0) \cup (0, \infty)$

Determine the end behavior of $f(x) = \frac{1}{x}$.

Graph

First, complete the tables.

going to the right

x Increases without Bound	
x	$f(x) = \frac{1}{x}$
100	0.01
1000	0.001
10,000	0.0001

going to the left

x Decreases without Bound	
x	$f(x) = \frac{1}{x}$
−100	−0.01
−1000	−0.001
−10,000	−0.0001

Next, summarize the results.

- As $x \to +\infty$, $f(x) \to$ 0.
- As $x \to -\infty$, $f(x) \to$ 0.

Right

Examine the behavior of $f(x) = \frac{1}{x}$ near $x = 0$, and determine what this means for the graph of the function.

First, complete the tables.

x Approaches 0 from the Positive Direction	
x	$f(x) = \frac{1}{x}$
0.01	100
0.001	1000
0.0001	10,000

Right

x Approaches 0 from the Negative Direction	
x	$f(x) = \frac{1}{x}$
−0.01	−100
−0.001	−1000
−0.0001	−10,000

Next, summarize the results.

- As $x \to 0^+$, $f(x) \to$ $+\infty$.
- As $x \to 0^-$, $f(x) \to$ $-\infty$.

Asymptote Behavior

The behavior of $f(x) = \frac{1}{x}$ near $x = 0$ indicates that the graph of $f(x)$ approaches, but does not cross, the [x-axis/y-axis], so that axis is also an asymptote for the graph.

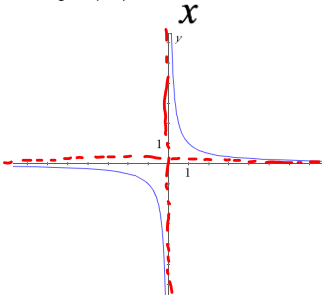
State the range of $f(x) = \frac{1}{x}$.

The function takes on all real numbers except 0, so the function's range is as follows:

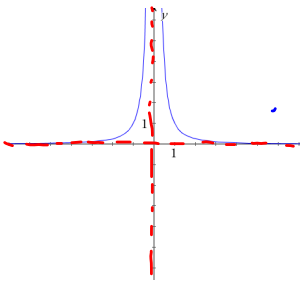
- As an inequality: $y <$ ___ or $y >$ ___
- In set notation: $\{y \mid y \neq$ ___$\}$
- In interval notation (where the symbol $\cup$ means *union*): $(-\infty, 0) \cup (0, +\infty)$

Look at the following Graphs $f(x) = \frac{1}{x}$ and $f(x) = \frac{1}{x^2}$ and compare. What is going on?

$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x^2}$



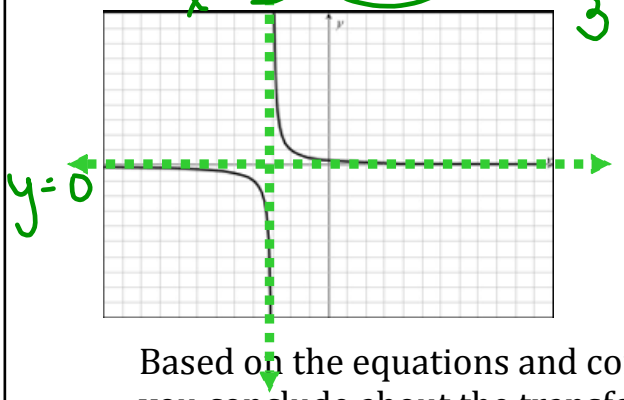
Similarity:
- Right ends same $x \to +\infty$, $y \to 0$
- H. Asymptote is at $y = 0$
- V. Asymptote at $x = 0$
- Domain: $(-\infty, 0) \cup (0, \infty)$

Difference:
- Left side is positive for $\frac{1}{x^2}$
- Ranges:
  $\frac{1}{x}$: $(-\infty, 0) \cup (0, \infty)$
  $\frac{1}{x^2}$: $(0, \infty)$

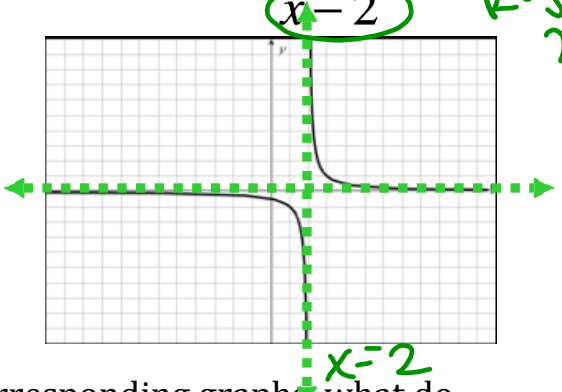
Look at the following graphs and the parent function from your function booklet and answer the question below.

$\frac{1}{x}$

$f(x)=\frac{1}{x+3}$ Shift left 3

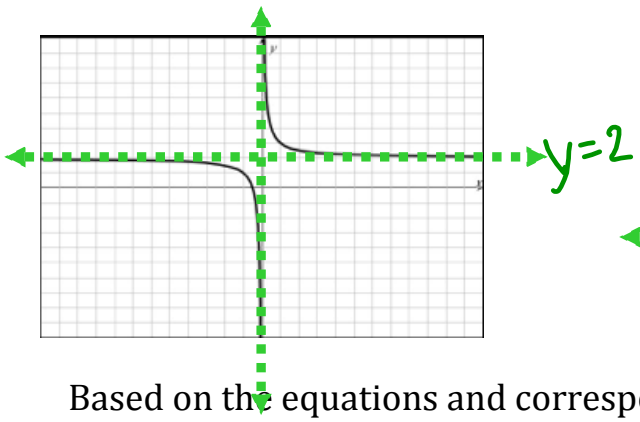


$f(x)=\frac{1}{x-2}$ Shift Right 2

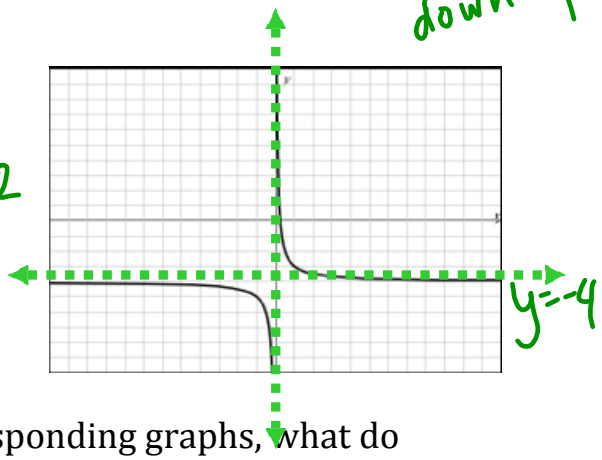


Based on the equations and corresponding graphs, what do you conclude about the transformations?

$f(x)=\frac{1}{x}+2$ Shift up 2

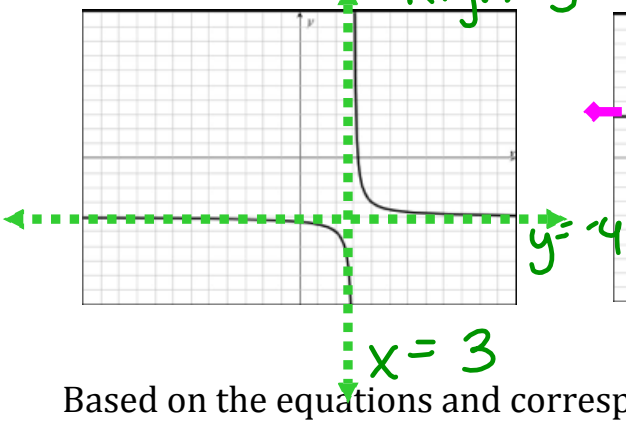


$f(x)=\frac{1}{x}-4$ Shift down 4

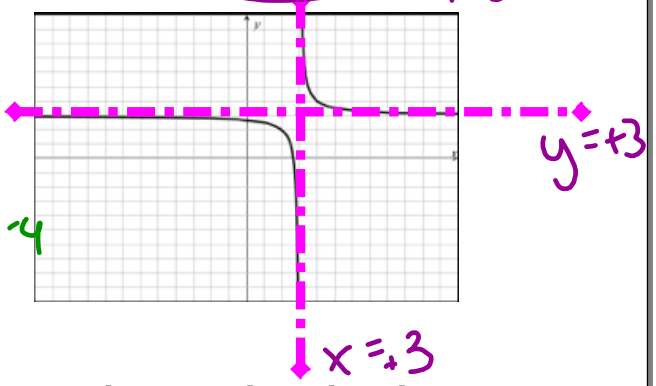


Based on the equations and corresponding graphs, what do you conclude about the transformations?

$f(x)=\frac{1}{x-3}-4$ Down 4, Right 3

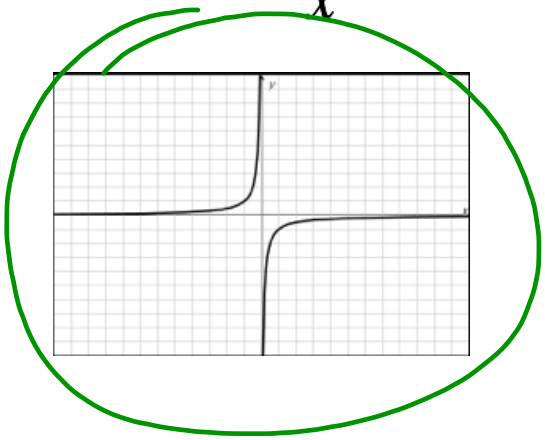


$f(x)=\frac{1}{x-3}+3$ ↑3 →3

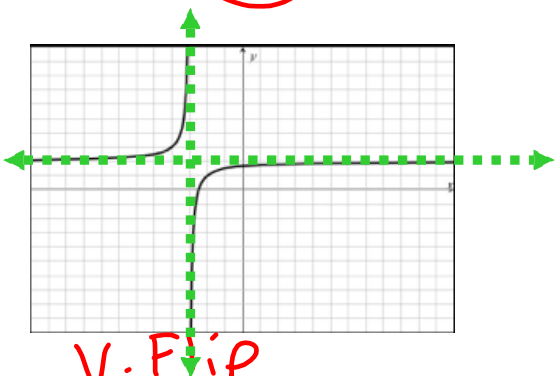


Based on the equations and corresponding graphs, what do you conclude about the transformations?

$f(x)=-\frac{1}{x}$ V. Flip



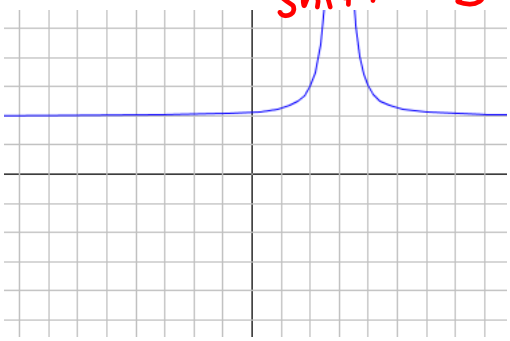
$f(x)=-\frac{1}{x+3}+2$



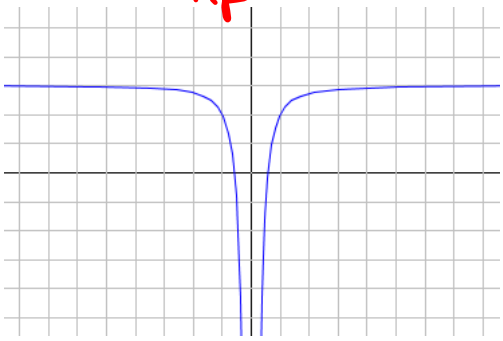
V. Flip
Shift Left 3
Up 2

Based on the equations and corresponding graphs, what do you conclude about the transformations?

$f(x)=\dfrac{1}{(x-3)^2}+2$ ↑2 Shift →3



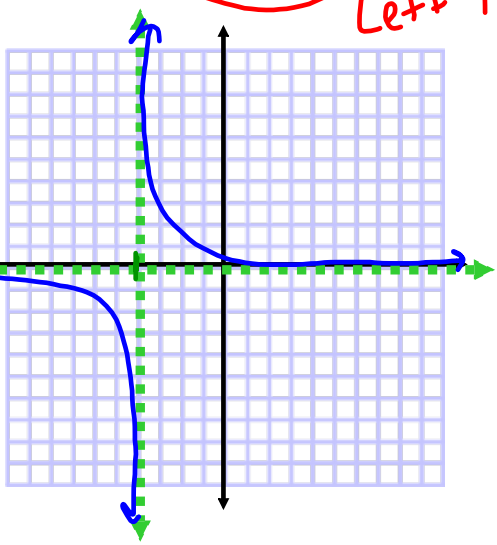
$f(x)=-\dfrac{1}{x^2}+3$ flip



Based on the equations and corresponding graphs, what do you conclude about the transformations?

Sketch a graph and analyze of the following.

$f(x)=\dfrac{1}{x+4}$ Shift Left 4



Domain: $(-\infty,-4)\cup(-4,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$

V Asymptote: $x=-4$

H Asymptote: $y=0$

Increasing: None

Decreasing: $(-\infty,-4)\cup(-4,\infty)$

End Behavior:

As $x\to-\infty$, $y\to 0$ H.A.

As $x\to+\infty$, $y\to 0$

Asymptote behavior:

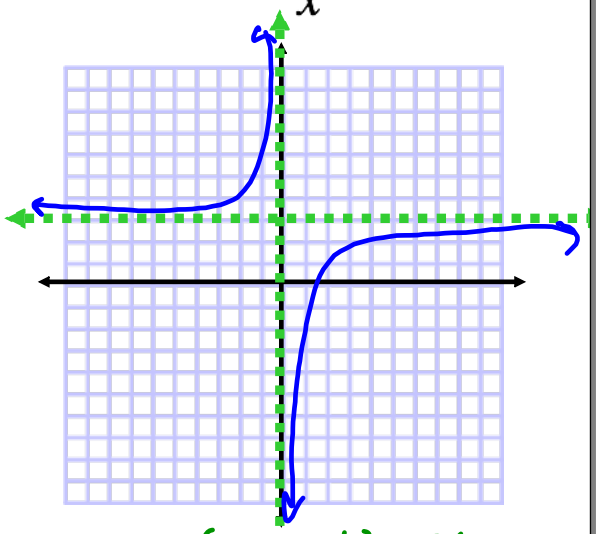
As $x\to-4^+$, $y\to+\infty$

As $x\to-4^-$, $y\to-\infty$

V.A

Sketch a graph and analyze of the following.

$$f(x) = -\frac{1}{x} + 3$$



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 3) \cup (3, \infty)$

V Asymptote: $x = 0$

H Asymptote: $y = 3$

Increasing: $(-\infty, 0) \cup (0, \infty)$

Decreasing: None

End Behavior:

As $x \to -\infty$, $y \to 3$

As $x \to +\infty$, $y \to 3$

V. Asymptote behavior:

As $x \to 0^+$, $y \to -\infty$

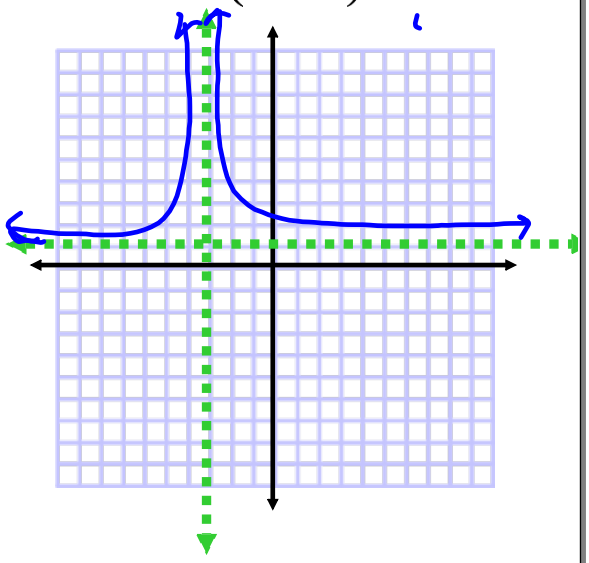
As $x \to 0^-$, $y \to +\infty$

Transformations:

- V. Flip
- Up 3

Sketch a graph and analyze of the following.

$$f(x) = \frac{1}{(x+3)^2} + 1$$



Domain: $(-\infty, -3) \cup (-3, \infty)$

Range: $(1, \infty)$

V Asymptote: $x = -3$

H Asymptote: $y = +1$

Increasing: $(-\infty, -3)$

Decreasing: $(-3, \infty)$

End Behavior:

Asymptote behavior:

Based on the conclusions you made, work with a partner to write an equation based on the following graphs.