5-1 Radical Review

- I can simplify radicals
- I can perform operations with radicals
- I know and can convert between radicals and fractional exponents

5-1 Radical Review

Definition
*n*th root

$$\sqrt[n]{b} = a \quad \text{means} \quad b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0.
- if $n \geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{\ }$ is called the <u>radical</u>

n is called the <u>index number</u>
b is called the <u>radicand</u>

if there is no index, it is <u>2</u>

Simplify

$5\sqrt[3]{24}$

$12 \cdot 2$, $6 \cdot 2$, $3 \cdot 2$

$5 \cdot 2\sqrt[3]{3}$

$\boxed{10\sqrt[3]{3}}$

$\sqrt[4]{20}$

$4 \cdot 5$, $2 \cdot 2$

$\sqrt[4]{20}$

$\sqrt[2]{128x^2}$

$\boxed{2}$ 64 $x \cdot x$

$8 \cdot 8$, $4 \; 2 \; 2 \; 4$, $2 \; 2$, $2 \; 2$

$2 \cdot 2 \cdot 2 \cdot x\sqrt{2}$

$\boxed{8x\sqrt{2}}$

$\sqrt{200a^2}$ $a \cdot a$

$2 \cdot 100$, $2 \cdot 50$, $\boxed{2}$ 25, $5 \; 5$

$2 \cdot 5 \cdot a\sqrt{2}$

$\boxed{10a\sqrt{2}}$

$\sqrt[4]{40}$

$4 \cdot 10$, $2 \; 2 \; 2 \; 5$

$\boxed{\sqrt[4]{40}}$

$4\sqrt[3]{54}$

$9 \cdot 6$, $3 \; 3 \; 3 \; 2$

$4 \cdot 3\sqrt[3]{2}$

$\boxed{12\sqrt[3]{2}}$

$\sqrt[3]{128x^6y^{10}}$

$2 \cdot 64$, $2 \cdot 32$, $8 \cdot 4$, $2 \; 4$, $2 \; \boxed{2}$, $2 \; 2$

$x \cdot x \cdot x \cdot x \cdot x \cdot x$

$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

$y^3 \cdot y^3 \cdot y^3 \cdot y^1$

$2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y\sqrt[3]{2y}$

$\boxed{4x^2y^3\sqrt[3]{2y}}$

$\sqrt[4]{16a^5b^{11}}$

$4 \cdot 4$, $2 \; 2 \; 2 \; 2$

$a^4 \; \boxed{a}$

$b^4 \cdot b^4 \cdot b^3$

$2ab^2\sqrt[4]{ab^3}$

Raise each of the following to the $\frac{1}{2}$ power.

$$1, 4, 9, 16, 25, 36$$

$$a^{\left(\frac{1}{2}\right)} = \underline{\sqrt[2]{a}}$$

index #

$16^{1/2}$: 16^(1/2)

Raise each of the following to the $\frac{1}{3}$ power. $1, 8, 27, 64, 125, 216$

$$a^{\left(\frac{1}{3}\right)} = \underline{\sqrt[3]{a}}$$

Fractional exponent

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

n is an integer bigger than or equal to 2

exponent

index # for radical

$$a^{\left(\frac{2}{3}\right)} = \underline{\sqrt[3]{a^2}} = \left(\sqrt[3]{a}\right)^2$$

$$a^{\left(\frac{m}{n}\right)} = \underline{\sqrt[n]{a^m}} = \left(\sqrt[n]{a}\right)^m$$

Write each of the following as a radical and simplify, if possible.

$9^{\frac{1}{2}}$

$\sqrt[2]{9} = 3$

3 3

$(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64} = -2 \cdot 2 \cdot 2$

$= -4$

-8 8

4 -2 4 2

-2 -2 2 2

$100^{\frac{1}{2}}$

$\sqrt[2]{100} = \boxed{10}$

10 10

$-100^{\frac{1}{2}}$

$z^{\frac{1}{2}}$

Rewrite in exponent form

$\sqrt[7]{x}$

$\boxed{x^{1/7}}$

$\sqrt[4]{b}$

$b^{1/4}$

$\sqrt[12]{r} = r^{1/12}$

$\sqrt[5]{d} = d^{1/5}$

Adding, Subtracting, and Multiplying Radical expressions

Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $n \geq 2$ is an integer, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

- multiply insides (if index # is the same)
- multiply outsides
- simplify

Multiply and Simplify Assuming all variables are greater than or equal to zero.

$$\sqrt{3} \cdot \sqrt{15}$$

$\sqrt{3 \cdot 15} = \sqrt{45}$

$3 \quad 15$

$3 \quad 5$

$3\sqrt{5}$

$$3\sqrt[3]{4x} \cdot \sqrt[3]{2x^4}$$

$3\sqrt[3]{4 \cdot 2 \cdot x \cdot x^4}$

$2 \quad 2$

x^3

$x \cdot x$

$\cancel{3} \cdot 2x\sqrt[3]{x^2}$

$\boxed{6x\sqrt[3]{x^2}}$

$$\sqrt[4]{27a^2b^5} \cdot \sqrt[4]{6a^3b^6}$$

Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, $n \geq 2$ is an integer, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Simplify Assuming all variables are greater than or equal to zero.

$$\frac{\sqrt{24a^3}}{\sqrt{6a}} = \sqrt{\frac{24a^3}{6a}} : \frac{\cancel{a}\cdot a\cdot a}{\cancel{a}}$$

$$\sqrt{4a^2} = 2a$$

$$\frac{-2\sqrt[3]{54a}}{\sqrt[3]{2a^4}}$$

$$-2\sqrt[3]{\frac{54a}{2a^4}}$$

$$-2\sqrt[3]{\frac{27a}{a^4}}$$

$$= -2\sqrt[3]{\frac{27}{a^3}}$$

$$-2\sqrt[3]{\frac{3\cdot 3\cdot 3}{a\cdot a\cdot a}}$$

$$\frac{-2\cdot 3}{a} = \frac{-6}{a}$$

$$\frac{\sqrt[3]{-375x^2y}}{\sqrt[3]{3x^{+1}y^7}}$$

$$\sqrt[3]{\frac{-375x^2y}{3xy^7}} = \sqrt[3]{\frac{-125x}{y^6}}$$

$-125 = -5\cdot 25$, $25 = -5\cdot -5$

$y^6 = y^3\cdot y^3$

$$\frac{-5}{y^2}\sqrt[3]{x}$$

$$\frac{-5\sqrt[3]{x}}{y^2}$$

Add or subtract as indicated. Assume all variables are real numbers greater than or equal to zero

$$3x\sqrt{20x} - 7\sqrt{5x^3}$$

$2 \cdot 10$, $2 \cdot 5$, $x \cdot x \cdot x$

$$6x\sqrt{5x} - 7x\sqrt{5x}$$

$$\boxed{-1x\sqrt{5x}}$$

$$3\sqrt{5} + 7\sqrt{13}$$

$$\boxed{3\sqrt{5} + 7\sqrt{13}}$$

$$4\sqrt{14} - 3\sqrt{8}$$

$2 \cdot 7$, $2 \cdot 4$, $2 \cdot 2$

$$\boxed{4\sqrt{14} - 6\sqrt{2}}$$

$$-5x\sqrt[3]{54x} + 7\sqrt[3]{2x^4}$$

$2 \cdot 27$, $3 \cdot 9$, $3 \cdot 3$, $x^3 \cdot x$

$$-5(3)x\sqrt[3]{2x} + 7x\sqrt[3]{2x}$$

$$-15x\sqrt[3]{2x} + 7x\sqrt[3]{2x}$$

$$\boxed{-8x\sqrt[3]{2x}}$$

27) $8^{1/3}$

29) $27^{4/3}$

30) $\dfrac{7}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \boxed{\dfrac{7\sqrt{13}}{13}}$

$\sqrt{13 \cdot 13}$