5-3 Solving Radical Equations

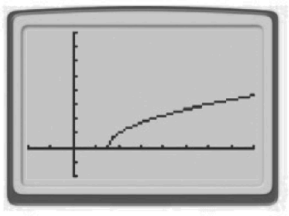
Objectives:

1. I can solve radical equations and check for extraneous solutions.

2. I can manipulate literal equations.

Remember that you can graph the two sides of an equation as separate functions to find solutions of the equation: a solution is any x-value where the two graphs intersect.

The graph of $y = \sqrt{x-3}$ is shown on a calculator window of $-4 \le x \le 16$ and $-2 \le y \le 8$. Reproduce the graph on your calculator. Then add the graph of $y = 2$.



How many solutions does the equation $\sqrt{x-3} = 2$ have? 1 How do you know?

The graphs cross once

On your calculator, replace the graph of $y = 2$ with the graph of $y = -1$.

How many solutions does the equation $\sqrt{x-3} = -1$ have? 0 How do you know?

They never cross

Find the solution graphically

y_1: $(x+5)^{\frac{1}{2}} - 2$ $=$ y_2: 1

$x = 4$

y_1: $2 + \sqrt{x+10}$ $=$ y_2: x

$x = 6$

$$(x+6)^{\frac{1}{2}} - (2x-4)^{\frac{1}{2}} = 0$$

$\sqrt{x+6} - \sqrt{2x-4} = 0$

$x = 10$

Solve the following, check for extraneous solutions

$(2\sqrt{x})^2 = (3\sqrt{x-2})^2$

$(2\sqrt{x})(2\sqrt{x})$

$4x = 9(x-2)$

$4x = 9x - 18$

$-9x \quad -9x$

$-5x = -18$

$x = \frac{-18}{-5} = 3.6$

$\sqrt{2x+5} + 4 = 3$

$-4 \quad -4$

$(\sqrt{2x+5})^2 = (-1)^2$

$2x + 5 = 1$

$-5 \quad -5$

$\frac{2x}{2} = \frac{-4}{2}$

$x = -2$ — No Solution

Check:

$\sqrt{2(-2)+5} + 4 = 3$

$\sqrt{-4+5} + 4 = 3$

$\sqrt{1} + 4 = 3$

$1 + 4 = 3$

$5 \neq 3$ X

$(\sqrt{5x-11})^2 = (x-1)^2$

$5x - 11 = (x-1)(x-1)$

$5x - 11 = x^2 - 1x - 1x + 1$

$5x - 11 = x^2 - 2x + 1$

$+11 \qquad +11$

$5x = x^2 - 2x + 12$

$-5x \qquad -5x$

$0 = x^2 - 7x + 12$

$0 = (x-3)(x-4)$

$x = 3, 4$

Check by plugging in!

$\sqrt{5(3)-11} = 3-1$

$\sqrt{15-11} = 2$

$\sqrt{4} = 2$

$2 = 2$ ✓

$\sqrt{5(4)-11} = 4-1$

$\sqrt{20-11} = 3$

$\sqrt{9} = 3$

$3 = 3$ ✓

Example 2 Solve the equation.

$\sqrt[3]{x+2}+7=5$

$-7 \quad -7$

$(\sqrt[3]{x+2})^3=(-2)^3$

$x+2=-8$

$-2 \quad -2$

$x=-10$

Check:

$\sqrt[3]{-10+2}+7=5$

$\sqrt[3]{-8}+7=5$

$-2+7=5$

$5=5$ ✓

$2(x-50)^{\frac{1}{3}}=-10$

$\frac{2\sqrt[3]{x-50}}{2}=\frac{-10}{2}$

$((x-50)^{\frac{1}{3}})^3=(-5)^3$

$x-50=-125$

$+50 \quad +50$

$x=-75$

Check:

$2(-75-50)^{1/3}=-10$

$2(-125)^{1/3}=-10$

$2(-5)=-10$

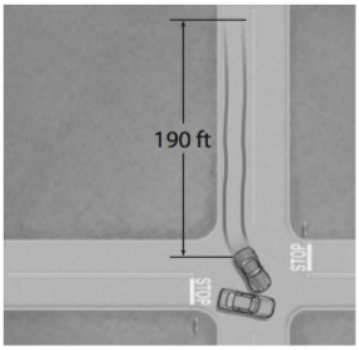
$-10=-10$ ✓

Solve the following:

$$\sqrt[3]{x-5}=\sqrt[3]{7-x}$$

$$\sqrt[3]{x+2}=\sqrt[3]{x+3}$$

Driving The speed s in miles per hour that a car is traveling when it goes into a skid can be estimated by using the formula $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the length of the skid marks in feet.



After an accident, a driver claims to have been traveling the speed limit of 55 mi/h. The coefficient of friction under the conditions at the time of the accident was 0.6, and the length of the skid marks is 190 feet. Is the driver telling the truth about the car's speed? Explain.

s d f

Use the formula to find the length of a skid at a speed of 55 mi/h. Compare this distance to the actual skid length of 190 feet.

$55 = \sqrt{30(0.6)(190)}$

$55 = \sqrt{3420}$ He's lying!

$55 = 58.48$

Your Turn

9. Biology The trunk length (in inches) of a male elephant can be modeled by $l = 23\sqrt[3]{t} + 17$, where t is the age of the elephant in years. If a male elephant has a trunk length of 100 inches, about what is his age?

t = age

$100 = 23\sqrt[3]{t} + 17$

$-17 \quad -17$

$\frac{83}{23} = \frac{23\sqrt[3]{t}}{23}$

$(3.6)^3 = \sqrt[3]{t}^3$

$47 = t$

46.99