④ $\frac{3x+6}{2x-4}$

$(3x+6) \div (2x-4)$

5-3: Graphing Rational Functions

Objectives:

1. I can find the x and y intercepts of a rational function
2. I can find the vertical and horizontal asymptotes of a rational function
3. I can find the holes of a rational function
4. I can analyze a graph of a rational function
5. I can graph a rational function by hand

X and Y Intercepts

Y intercepts, x = 0

$$f(x)=\frac{3x-12}{x^2-5x-6} = \frac{3(0)-12}{0^2-5(0)-6} = \frac{-12}{-6} = 2$$

y-int: (0, 2)

(0, y)

X intercepts, y = 0

Set top = 0

$$f(x)=\frac{3x-12}{x^2-5x-6}$$

$3x - 12 = 0$

$+12 \quad +12$

$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

x-int: (4, 0)

(x, 0)

Find the x and y intercepts of the following functions:

$$f(x)=\frac{(x-3)(x+1)}{x+2}$$

x-int: (3, 0) (−1, 0)

$(x-3)(x+1)=0$

3 −1

y-int: (0, −3/2)

$$\frac{(0-3)(0+1)}{0+2} = \frac{(-3)(1)}{2} = \frac{-3}{2}$$

$$f(x)=\frac{3x-5}{(x-2)(x-3)}$$

Review of Vertical Asymptotes

$$f(x)=\frac{2}{x+3}$$

VA

$x = -3$

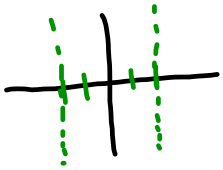
Set the denominator = 0, then solve for x

VA: Vertical Line the graph approaches and doesn't touch

Find the vertical asymptotes:

a. $y=\dfrac{3x-5}{(x-2)(x+2)}$

$x = +2, -2$



b. $y=\dfrac{2x^3}{x-5}$

$x = 5$

c. $y=\dfrac{5x}{x+2}$

$x = -2$

Asymptotes:

check for holes before VA!! (by reducing the fraction if possible)

$x \neq 2$

$$f(x) = \frac{(x-3)(x-2)}{(x-2)} = x-3$$

vertical (VA): caused by dividing by 0
the graph approaches $-\infty$ *or* ∞
on each side of the asymptote
to find the asymptote set den = 0 and solve

Hole: cancel out the factor that matches on top & bottom
and where that factor = 0
creates a hole

Identify any holes, then find all vertical asymptotes

$$f(x) = \frac{(x-3)(x+3)}{(x-2)(x-3)} = \frac{(x+3)}{(x-2)}$$

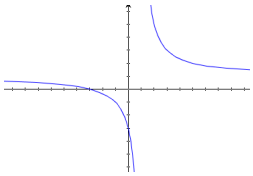
Hole: $x = 3$

VA: $x = 2$

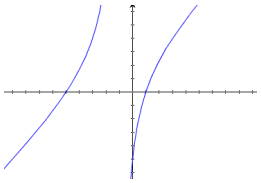
Horizontal Asymptotes

Look at the graphs, see if you can find the horizontal asymptote. Are there any patterns?

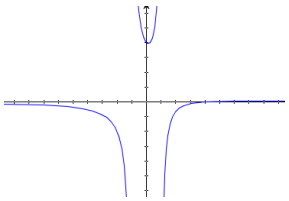
$f(x)=\dfrac{x+3}{x-1}$



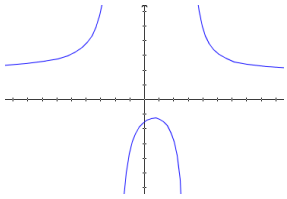
$f(x)=\dfrac{(x+5)(x-1)}{x+1}$



$f(x)=\dfrac{x-4}{(x+1)(x-1)}$



$f(x)=\dfrac{2x^2-3x+9}{x^2-x-6}$



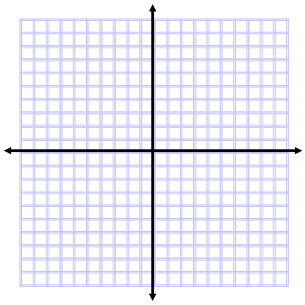
end behavior: (horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the numerator and denominator if:

top heavy (OA):
bottom heavy (HEB): y = 0
equal (HA): divide coefficients

Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

$f(x) = \dfrac{-3}{x-1}$



$f(x) = \dfrac{3x-7}{x-2} = 0$

$3x - 7 = 0$

$+7 \quad +7$

$\dfrac{3x}{3} = \dfrac{7}{3}$

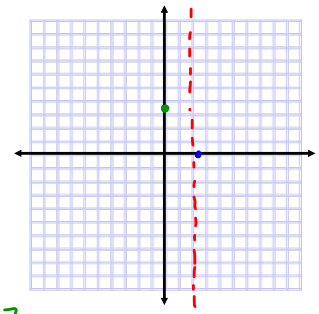
$x = 7/3$

Holes: None

VA: $x = 2$

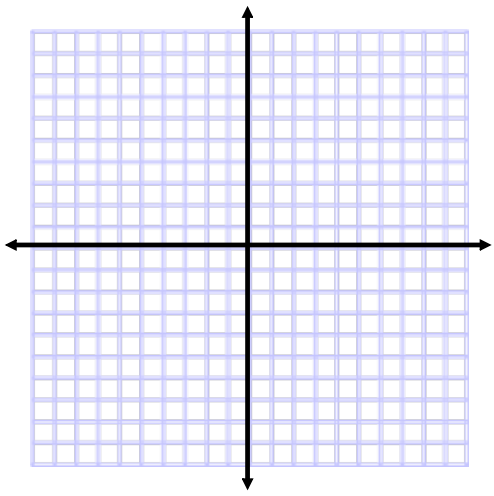
x-int: $(7/3, 0)$

y-int: $(0, 7/2)$ $\dfrac{3(0)-7}{0-2} = \dfrac{-7}{-2} = \dfrac{7}{2}$



Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

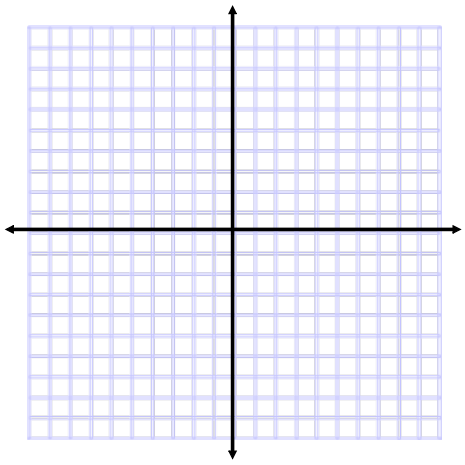
$f(x) = \dfrac{3x-2}{x-1}$



Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x)=\frac{x-1}{(x-4)(x+3)}$$

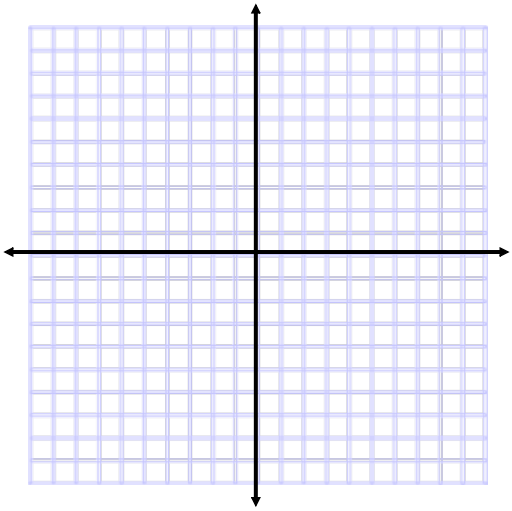
Domain
Range
x-intercepts
y-intercepts
VA
HA
Asymptote Behavior



End Behavior

Graph and analyze $f(x)=\frac{4x+7}{x+4}$

Domain
Range
x-intercepts
y-intercepts
VA
HA

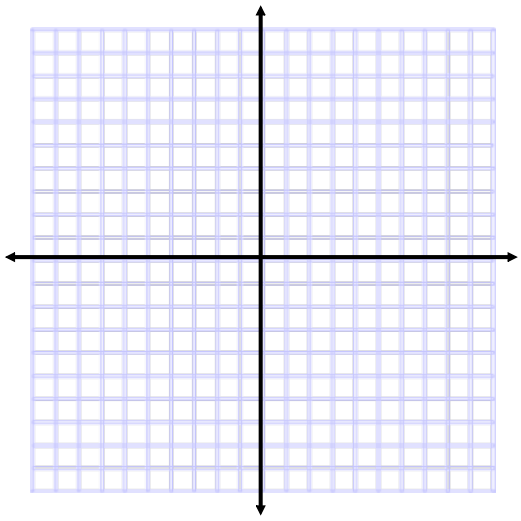


Asymptote Behavior

End Behavior

Graph and Analyze $f(x)=\dfrac{x+1}{(x+3)(x-4)}$

Domain
Range
x-intercepts
y-intercepts
VA
HA

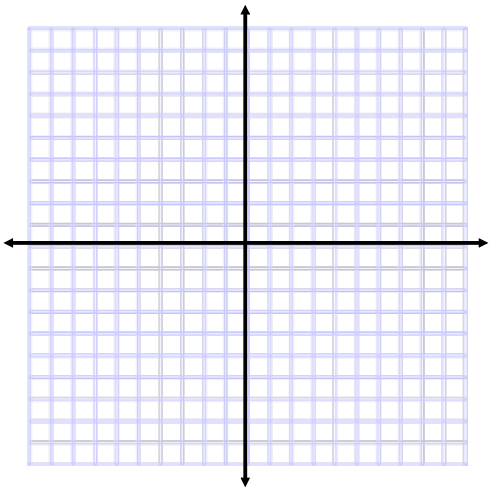


Asymptote Behavior

End Behavior

Graph and analyze $f(x)=\dfrac{4x-4}{x+2}$

Domain
Range
x-intercepts
y-intercepts
VA
HA



Asymptote Behavior

End Behavior