6-4 Inverse Functions

Objectives:
-I can find the inverse of a given function graphically and algebraically
-I can analyze the domain of a function and its inverse

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x).

(2, 5)
(5, 2)

Notation:

$$f^{-1}(x)$$

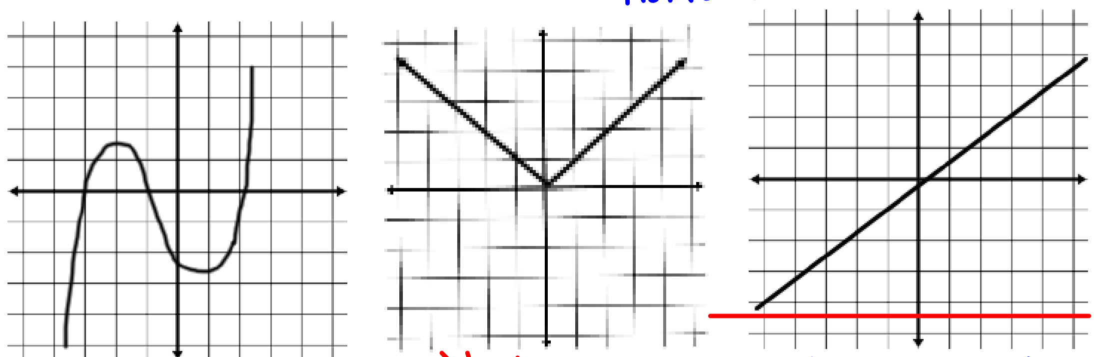
Represents the inverse of the function $f(x)$

Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

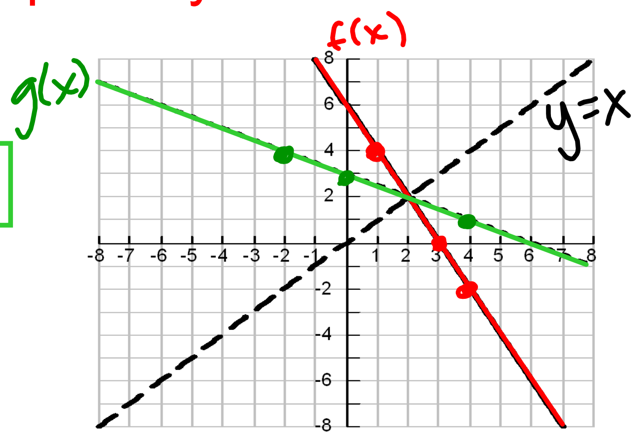
Horizontal line test



Not one-to-one Not one-to-one Yes, one-to-one

Inverses - graphically

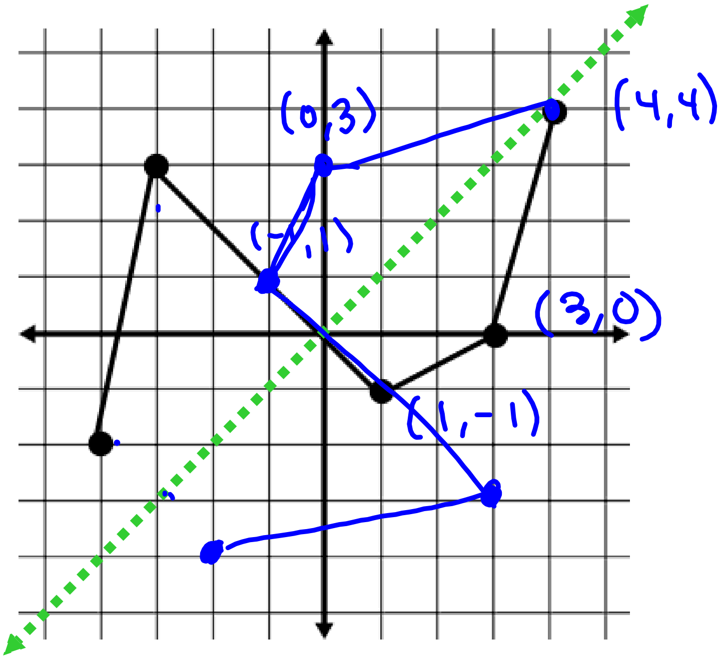
Show $\boxed{f(x) = 6 - 2x}$ and $\boxed{g(x) = \frac{6-x}{2}}$ are inverses graphically.



f(x):	(1,4)	(3,0)	(4,-2)
g(x):	(4,1)	(0,3)	(-2,4)

Graph the inverse of the graph. (Use y=x to find inverse points)

diagonal



To find the inverse equation of a function

1. Change *f(x)* to *y*.
2. Interchange *x* and *y*
3. Solve for *y*
4. Change new *y* to $f^{-1}(x)$

Find the inverse of each function

$\cancel{f(x)}^{y} = x^2 + 1$

$x = y^2 + 1$
$\quad -1 \qquad -1$

$\sqrt{x-1} = \sqrt{y^2}$

$\sqrt{x-1} = y$

$\boxed{f^{-1}(x) = \sqrt{x-1}}$

1. $h\cancel{(x)}^{y} = 2x^3 + 3$

2. $x = 2y^3 + 3$
$\quad -3 \qquad -3$

3. $\frac{(x-3)}{2} = \frac{\cancel{2}y^3}{\cancel{2}}$

$\sqrt[3]{\frac{(x-3)}{2}} = \sqrt[3]{y^3}$

$\sqrt[3]{\frac{x-3}{2}} = y$

4. $\boxed{h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}}$

1. $\cancel{g(x)}^{\,y} = \sqrt[3]{x} - 3$

2. $x = \sqrt[3]{y} - 3$

$+3 \qquad +3$

3. $(x+3)^3 = \cancel{\sqrt[3]{y}^{\,3}}$

4. $\boxed{g^{-1}(x) = (x+3)^3} = y$

$\cancel{g(x)}^{\,y} = \dfrac{x+1}{2x+3}$

$(2y+3)x = \dfrac{(y+1)\cancel{(2y+3)}}{\cancel{(2y+3)}}$

$2xy + 3x = y + 1$

$-y \qquad -y$

$2xy - y + 3x = 1$

$-3x \qquad -3x$

$2xy - y = -3x + 1$

$\dfrac{y(2x-1)}{(2x-1)} = \dfrac{-3x+1}{2x-1}$

$y = \boxed{\dfrac{(-3x+1)}{(2x-1)} = g^{-1}(x)}$

1. get rid of denominator by multiplyin

2. move y's to the same side

3. move anything w/out y to other side

4. Factor out a y

④ $y = \frac{2x-3}{x+1}$

$(y+1)x = \frac{2y-3}{\cancel{(y+1)}}\cancel{(y+1)}$

$xy + 1x = 2y - 3$
$\quad -2y \qquad -2y$

$xy - 2y + x = -3$
$\qquad\quad -x \quad -x$

$xy - 2y = -x - 3$

$\frac{y\cdot\cancel{(x-2)}}{\cancel{(x-2)}} = \frac{-x-3}{x-2}$

$\boxed{y = \frac{(-x-3)}{(x-2)}}$

1. What are the 4 steps to finding inverses?

1. Change f(x) to y
2. Switch x & y
3. Solve for y
4. Change y to $f^{-1}(x)$

2. Find the inverse of: $\cancel{f(x)} = 3x - 2$ y

$x = 3y - 2$
$+2 \qquad +2$

$\frac{x+2}{3} = \frac{\cancel{3}y}{\cancel{3}}$

$\boxed{f^{-1}(x) = \frac{x+2}{3}}$

3. Find the inverse of: $f(x) = \frac{x+2}{3}$

$3x = \frac{(y+2)}{\cancel{3}}\cancel{3}$

$3x = y + \cancel{2}$
$-2 \qquad \cancel{-2}$

$\boxed{f^{-1}(x) = 3x - 2}$

4. Find the inverse of: $f(x) = \frac{x+2}{x-3}$

$$\overset{y}{\cancel{f(x)}} = \frac{x+2}{x-3}$$

$$(y-3)x = \frac{y+2}{\cancel{(y-3)}}\cancel{(y-3)}$$

$$xy - 3x = y + 2$$

$$-y \qquad -y$$

$$xy - y \cancel{-3x} = 2$$

$$+3x \qquad +3x$$

$$xy - y = 3x + 2$$

$$\frac{y\cdot\cancel{(x-1)}}{\cancel{(x-1)}} = \frac{3x+2}{(x-1)}$$

$$f^{-1}(x) = \frac{3x+2}{(x-1)} = \frac{-3x-2}{1-x}$$

$$\frac{-\cancel{1}(3x+2)}{-\cancel{1}(x-1)}$$

Due at the end of class:

- White "warm-up" paper (4 questions)
- Factoring Practice Worksheet

$$-\frac{4}{3}y = \frac{-4y}{3}$$