6-4 Exponential Review

Objectives:

-I can apply exponential properties and use them

-I can model real-world situations using exponential functions

EXPONENTIAL FUNCTION

$$f(x) = a(b)^x$$

Exponent

Initial Value (y-intercept)

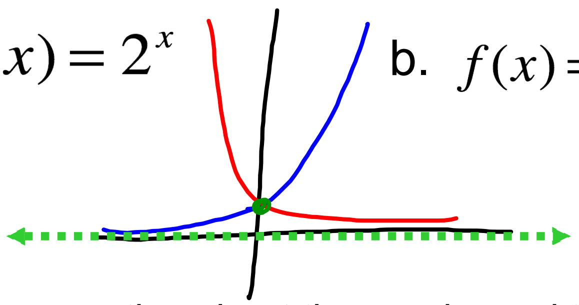
Base (Multiplier)

0th term

$x = 0$

Graph the following functions on a calculator and sketch.
Be sure to plot the y-intercept

a. $f(x) = 2^x$



b. $f(x) = \left(\frac{1}{2}\right)^x$

$.5^x$

What did you notice about the graphs and their equations?

- Both have y-int @ (0,1)
- Same shape → reflected
- 2^x increasing (growth)
- $(1/2)^x$ decreasing (decay)
- Horizontal asymptote @ y=0 (x-axis)

f(x) = a(b)^x

Exponential Growth and Decay

bigger than 1

When b>1, the function represents **exponential growth**

When 0<b<1, the function represents **exponential decay**

fraction less than 1

Determine whether each function represents growth or decay

a. $f(x)=13\left(\frac{1}{3}\right)^x$

$\frac{1}{3} < 1$

$\boxed{\text{Decay}}$

b. $g(x)=\left(\frac{3}{2}\right)^x$

$1.5 = \frac{3}{2} > 1$

$\boxed{\text{Growth}}$

Growth/Decay Equation

$$f(t)=a(1\pm r)^t$$

initial

rate

% as decimal

time (variable)

+ : increasing

- : decreasing

The population of Orem in 1950 was 4,000 and was increasing at a rate of 2.6% per year.

a) Predict the population of Orem in 1975 and 2000.

Starting year = 1950 $(t=0)$

$a = 4000$

$r = 2.6\% = .026$

$f(t) = 4000(1+.026)^t$

1975 : $t = 25$

2000 : $t = 50$

$4000(1.026)^{25} = 7599$

$4000(1.026)^{50} = 14{,}435$

b) Using your graphing calculator, predict when Orem's population will hit 200,000 people.

$$\underbrace{4000(1.026)^t}_{y_1} = \underbrace{200000}_{y_2}$$

$\underset{(t)}{x} = 152.4$ after 1950

year: 2102

On federal income tax returns, self employed people can depreciate the value of business equipment. Suppose a computer valued at \$2765 depreciates at a rate of 30% per year.

$$f(t) = a(1 \pm r)^t \quad -$$

a) Write an exponential equation to model this situation

$a = 2765$

$r = 30\% = 0.3$

$f(t) = 2765(1-0.3)^t$

b) How much will this computer be worth in 5 years?

$2765(0.7)^5 = \$464.71$

c) Use your graphing calculator to determine in how many years will the computer be worth \$350.

$$\underbrace{2765(1-.3)^t}_{y_1} = \underbrace{350}_{y_2}$$

$x = 5.8$ yrs

Compound Interest Formula

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

P is the principal (start)

r is the annual interest rate (% as a decimal)

n is the number of compounding periods per year

t is the time in years

annually: n=1
Semiannual: n=2
quarterly: n=4
monthly: n=12
daily: n=365

Write an equation then find the final amount for each investment.

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

a. $1000 at 8% compounded semiannually for 15 years

P   0.08   n=2

$t \leftarrow 15$ = $3243.40

$$A(t) = 1000\left(1+\frac{.08}{2}\right)^{(2t)}$$

$$1000\left(1+\frac{.08}{2}\right)^{(2\cdot 15)}$$

Using a calculator, determine how many years it will take for equation b to have a final amount of $4000.

The value e is called the natural base

The exponential function with base e, $f(x)=e^x$, is called the natural exponential function.

$$e \approx 2.71828182827$$

what you need to know is $e \approx 2.7$

Evaluate $f(x)=e^x$ for

a. $x = 2$

b. $x = \frac{1}{2}$

c. $x = -1$

Continuous Compounding Formula

If P dollars are invested at an interest rate r, that is compounded continuously, then the amount, A, of the investment at time t is given by

$$A(t) = Pe^{rt}$$

time

rate

Principal

A person invests \$1550 in an account that earns 4% annual interest compounded continuously. $A(t) = Pe^{rt}$

.04

a. Write an equation to represent this situation

$$A(t) = 1550e^{.04t}$$

b. Using a calculator, find when the value of the investment reaches \$2000.

An investment of $1000 earns an annual interest rate of 7.6%.

Compare the final amounts after 8 years for interest *compounded quarterly* and for interest *compounded continuously.*