

⑧ ⑨ 3. $\frac{5}{x^2 - 3x + 2} - \frac{1}{x-2} = \frac{x+6}{3x-3} \cdot \frac{(x-2)}{(x-2)}$
 $\frac{3(x-2)(x-1)}{3(x-1)}$

L(D: $\frac{3(x-2)(x-1)}{x \neq 2, 1}$) $(x+6)(x-2)$
 $15 + (-3x+3) = x^2 - 2x + 6x - 12$
 $\cancel{-3x+18} + \cancel{3x-18} = \cancel{x^2+4x-12} - 3x - 18$
 $0 = x^2 + 7x - 30$
 $0 = (x+10)(x-3)$
 $x = -10, 3$

⑦ $\frac{2x+6}{2} - \frac{6}{6} = \frac{x+4}{x+4}$
 $\frac{\cancel{2}(x+3)}{(x+4)} - \frac{2 \cdot 3}{(x+3)(x+4)} = (x+4) \cdot \cancel{6(x+3)}$
 L(D: $2 \cdot 3(x+3)(x+4)$)

6-4 Inverse Functions

Objectives:

- I can find the inverse of a given function graphically and algebraically
- I can analyze the domain of a function and its inverse

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x) .

Notation:

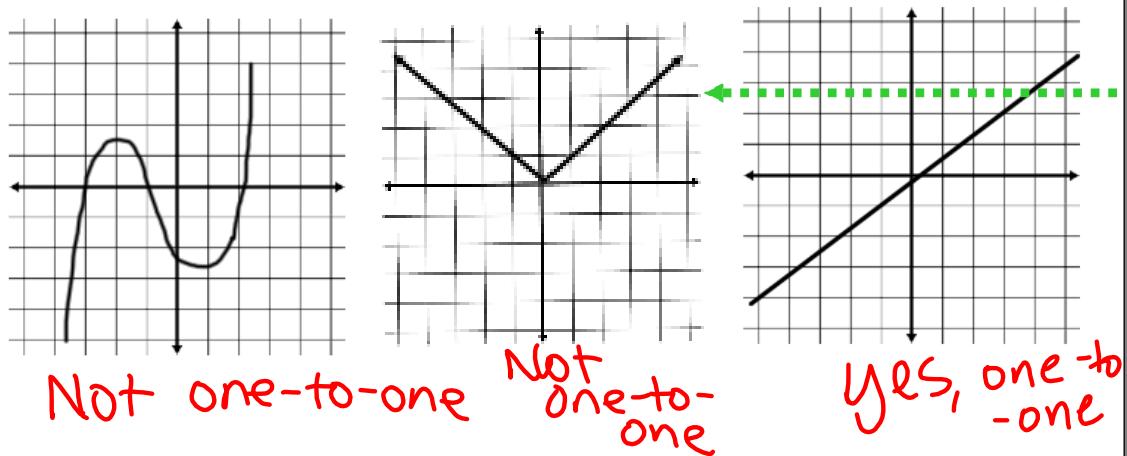
$$f^{-1}(x)$$

Represents the inverse of the function $f(x)$

Horizontal-Line Test

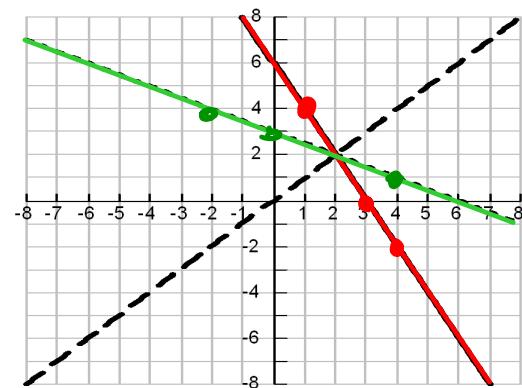
The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.



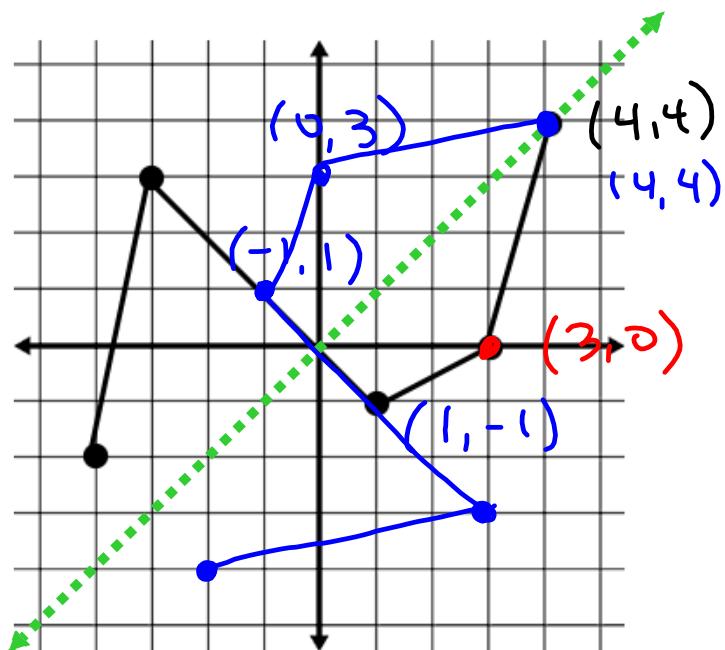
Inverses - graphically

Show $f(x) = 6 - 2x$ and $g(x) = \frac{6-x}{2}$ are inverses graphically.



$f(x)$:	$(1, 4)$	$(3, 0)$	$(4, -2)$
$g(x)$:	$(4, 1)$	$(0, 3)$	$(-2, 4)$

Graph the inverse of the graph. (Use $y=x$ to find inverse points)



reflect across
 $y=x$

• switching
x's & y's

To find the inverse equation of a function

1. Change $f(x)$ to y .
2. Interchange x and y
3. Solve for y
4. Change new y to $f^I(x)$

Find the inverse of each function

$$f(x) = x^2 + 1$$

$$\begin{array}{r} x = y^2 + 1 \\ -1 \end{array}$$

$$\pm\sqrt{x-1} = \sqrt{y^2}$$

$$\pm\sqrt{x-1} = y \quad f^{-1}(x)$$

$$f^{-1}(x) = \pm\sqrt{x-1}$$

$$h(x) = 2x^3 + 3$$

$$\begin{array}{r} x = 2y^3 + 3 \\ -3 \\ \hline x-3 = 2y^3 \\ \frac{x-3}{2} = y^3 \end{array}$$

$$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

$$g(x) = \sqrt[3]{x-3}$$

$$\begin{array}{r} x = \sqrt[3]{y} - 3 \\ +3 \\ \hline (x+3)^3 = y \end{array}$$

$$(x+3)^3 = y \quad g^{-1}(x) =$$

$$g(x) = \frac{x+1}{2x+3}$$

$$(2y+3)x = \frac{y+1}{(2y+3)}(2y+3)$$

$$2xy + 3x = y + 1$$

$$\frac{2xy + 3x}{-y} = \frac{y+1}{-y}$$

$$2xy - y + 3x = 1$$

$$2xy - y = -3x + 1$$

$$y \cdot (2x-1) = \frac{-3x+1}{2x-1}$$

$$f^{-1}(x) = \frac{-3x+1}{2x-1}$$

$$\textcircled{4} \quad (y+1)x = \frac{2y-3}{y+1} (y+1)$$

$$xy + x = \frac{2y-3}{y+1}$$

$$\frac{-2y}{-2y} \quad \frac{-2y}{-2y}$$

$$xy - 2y + x = -3$$

$$\frac{-x}{-x} \quad \frac{-x}{-x}$$

$$xy - 2y = -x - 3$$

$$\frac{y \cdot (x-2)}{(x-2)} = \frac{-x-3}{(x-2)}$$

$$f^{-1}(x) = \frac{-x-3}{(x-2)}$$