7-1 Rational Functions

Objectives:

- I can determine the domain, range, end behavior, and intervals of increasing and decreasing of rational functions.
- I can identify the transformation of a given function and sketch a graph
- I can write a rational equation given a graph.

State the domain of $f(x) = \frac{1}{x}$. $x \neq 0$

The function accepts all real numbers except 0, because division by 0 is undefined. So, the function's domain is as follows:

- As an inequality: $x < \square$ or $x > \square$
- In set notation: $\{x \mid x \neq \square\}$
- In interval notation (where the symbol $\cup$ means *union*):

$(-\infty, \boxed{0}) \cup (\boxed{0}, +\infty)$

Determine the end behavior of $f(x) = \frac{1}{x}$.

First, complete the tables.

x Increases without Bound	
x	$f(x) = \frac{1}{x}$
100	.01
1000	.001
10,000	.0001
$+\infty$	0

x Decreases without Bound	
x	$f(x) = \frac{1}{x}$
-100	$-.01$
-1000	$-.001$
$-10{,}000$	$-.0001$
$-\infty$	0

Next, summarize the results.

- As $x \to +\infty$, $f(x) \to$ ☐.
- As $x \to -\infty$, $f(x) \to$ ☐.

$$\lim_{x \to \infty} f(x) = 0 \qquad \lim_{x \to -\infty} f(x) = 0$$

Examine the behavior of $f(x) = \frac{1}{x}$ near $x = 0$, and determine what this means for the graph of the function.

First, complete the tables.

x Approaches 0 from the Positive Direction	
x	$f(x) = \frac{1}{x}$
0.01	$\frac{1}{0.01} = 100$
0.001	1000
0.0001	10000
	$+\infty$

x Approaches 0 from the Negative Direction	
x	$f(x) = \frac{1}{x}$
-0.01	-100
-0.001	-1000
-0.0001	-10000
	$-\infty$

Next, summarize the results.

- As $x \to 0^+$, $f(x) \to$ $\boxed{+\infty}$.
- As $x \to 0^-$, $f(x) \to$ $\boxed{-\infty}$.

The behavior of $f(x) = \frac{1}{x}$ near $x = 0$ indicates that the graph of $f(x)$ approaches, but does not cross, the [x-axis/y-axis], so that axis is also an asymptote for the graph.

$$\lim_{x \to 0^+} f(x) = +\infty \qquad \lim_{x \to 0^-} f(x) = -\infty$$

y-values

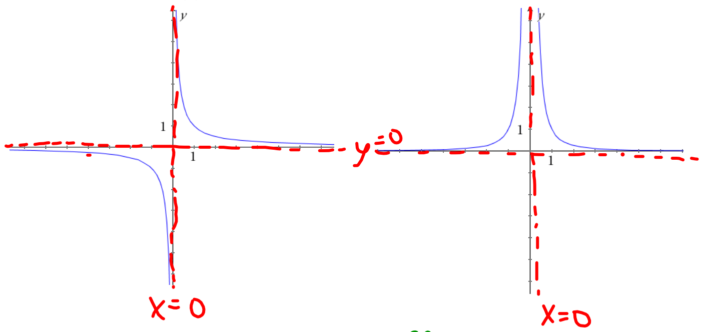
State the range of $f(x) = \frac{1}{x}$.

The function takes on all real numbers except $\underline{\ 0\ }$, so the function's range is as follows:

- As an inequality: $y < \square$ or $y > \square$
- In set notation: $\{y \mid y \neq \square\}$
- ★ In interval notation (where the symbol $\cup$ means *union*): $(-\infty, \boxed{0}) \cup (\boxed{0}, +\infty)$

Look at the following Graphs $f(x) = \frac{1}{x}$ and $f(x) = \frac{1}{x^2}$ and compare. What is going on?

$$f(x) = \frac{1}{x} \qquad\qquad f(x) = \frac{1}{x^2}$$

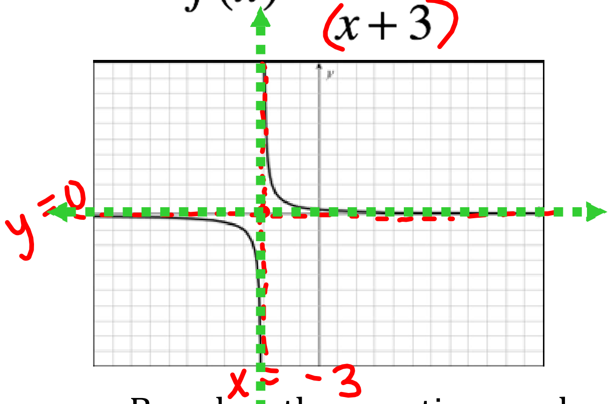


Similarities:
- Numerators are same (1)
- Top Right Corner (quadrant) same
- VA: $x=0$
- HA: $y=0$

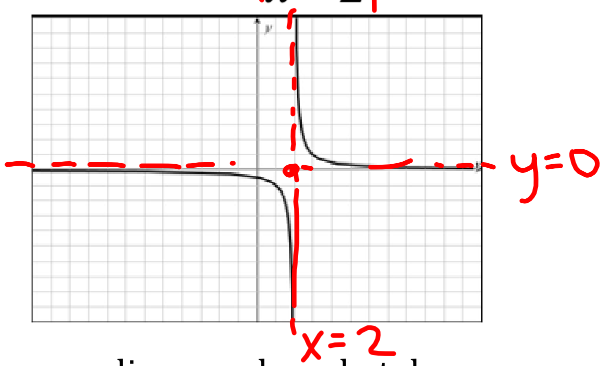
Differences:
- Left Corners are different
- Ranges

Look at the following graphs and the parent function from your function booklet and answer the question below.

$$f(x)=\frac{1}{(x+3)}$$



$$f(x)=\frac{1}{(x-2)}$$

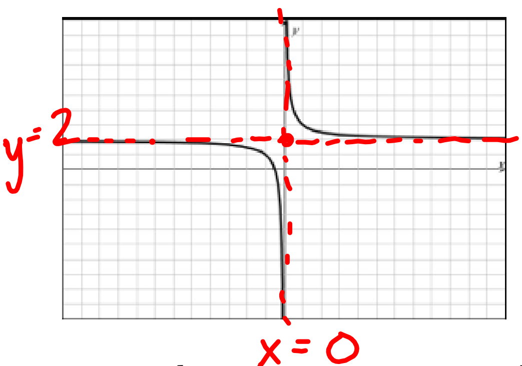


Based on the equations and corresponding graphs, what do you conclude about the transformations?

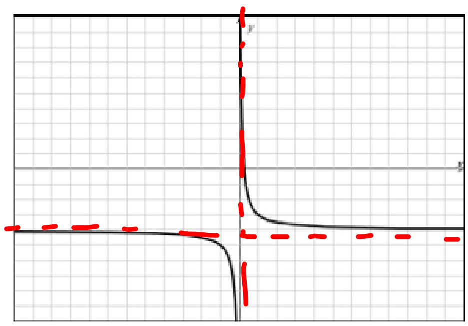
- Shift Left by 3

Shift Right by 2

$$f(x)=\frac{1}{x}+2$$



$$f(x)=\frac{1}{x}-4$$

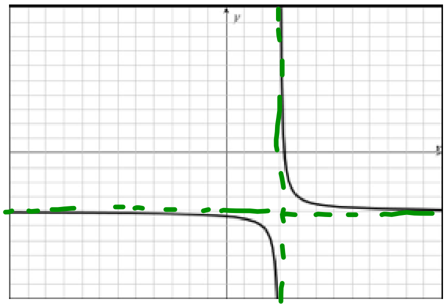


Based on the equations and corresponding graphs, what do you conclude about the transformations?

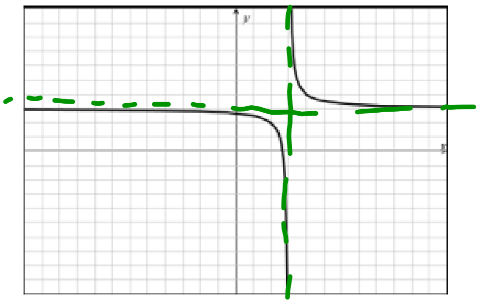
· Shifted up 2

Shifted down 4

$$f(x)=\frac{1}{x-3}-4$$



$$f(x)=\frac{1}{(x-3)}+3$$

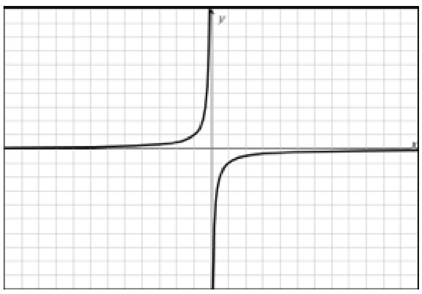


Based on the equations and corresponding graphs, what do you conclude about the transformations?

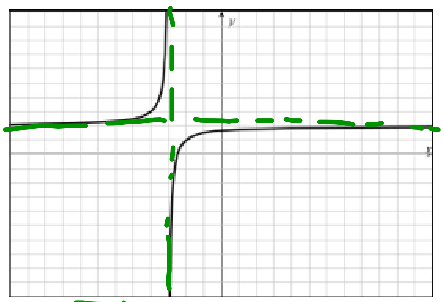
- Right 3 → VA
- Down 4 → HA

Right 3
Up 3

$$f(x)=-\frac{1}{x}$$



$$f(x)=-\frac{1}{x+3}+2$$



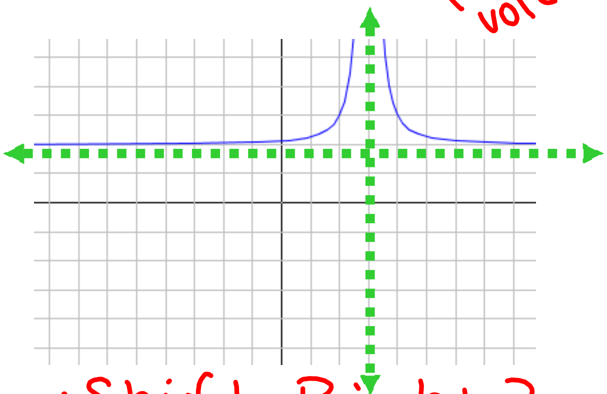
V. Flip

- V. Flip
- Shift Left 3 · Up 2

Based on the equations and corresponding graphs, what do you conclude about the transformations?

$$f(x)=\frac{1}{(x-3)^2}+2$$

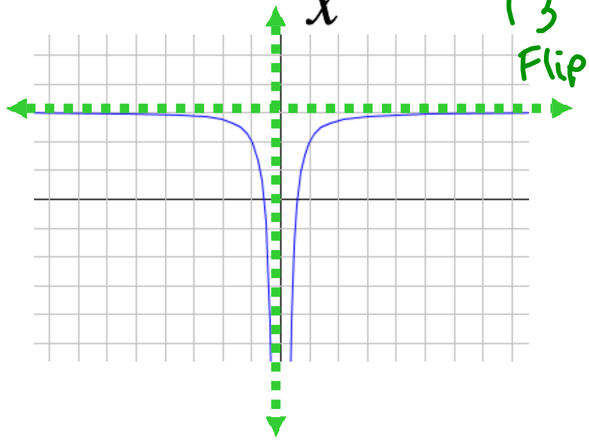
volcano



$$f(x)=-\frac{1}{x^2}+3$$

↑3

Flip



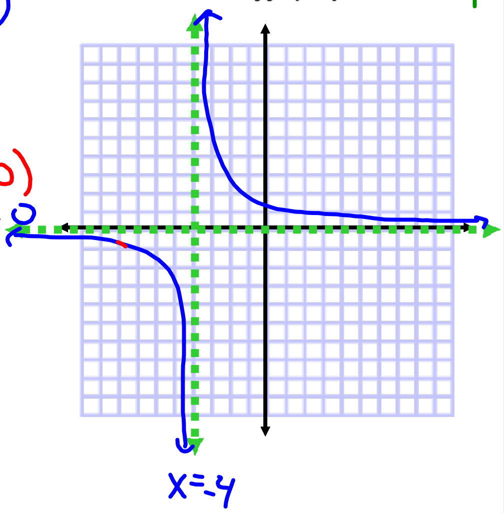
- Shift Right 3
- Shift up 2

Based on the equations and corresponding graphs, what do you conclude about the transformations?

Sketch a graph and analyze of the following.

$$f(x)=\frac{1}{x+4}$$

← 4



x Domain: $(-\infty,-4)\cup(-4,\infty)$

y Range: $(-\infty,0)\cup(0,\infty)$

V Asymptote: $x=-4$

H Asymptote: $y=0$

x's Increasing: None

Decreasing: $(-\infty,-4)\cup(-4,\infty)$

End Behavior:

$\lim_{x\to-\infty} f(x)=0$ ← HA

$\lim_{x\to\infty} f(x)=0$

Asymptote behavior:

Left $\lim_{x\to -4^-} f(x)=-\infty$

Right $\lim_{x\to -4^+} f(x)=+\infty$

VA

Sketch a graph and analyze of the following.

$$f(x) = -\frac{1}{x} + 3$$

V. Flip

↑3

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 3) \cup (3, \infty)$

V Asymptote: $x = 0$

H Asymptote: $y = 3$

x's Increasing: $(-\infty, 0) \cup (0, \infty)$

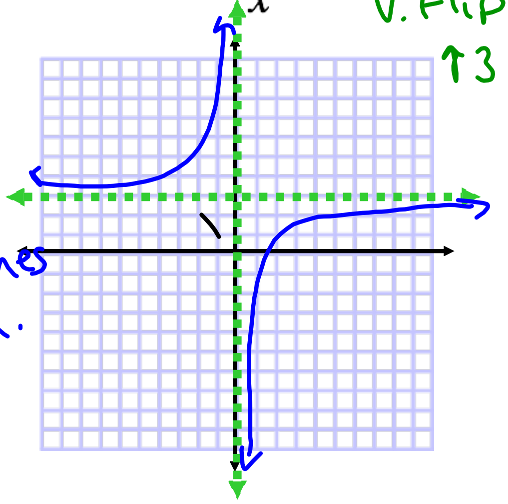
Decreasing: None

End Behavior:

$\lim_{x \to -\infty} f(x) = 3$

$\lim_{x \to +\infty} f(x) = 3$

Matches H.A.



V. Asymptote behavior:

Left $\lim_{x \to 0^-} f(x) = +\infty$

Right $\lim_{x \to 0^+} f(x) = -\infty$

V.A.

Sketch a graph and analyze of the following.

$$f(x) = \frac{1}{(x+3)^2} + 1$$

Domain:

Range:

V Asymptote:

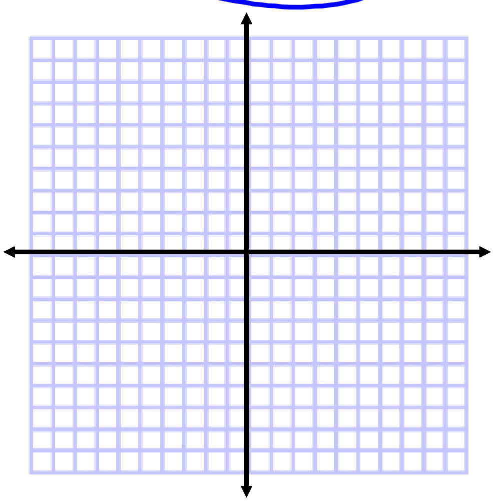
H Asymptote:

Increasing:

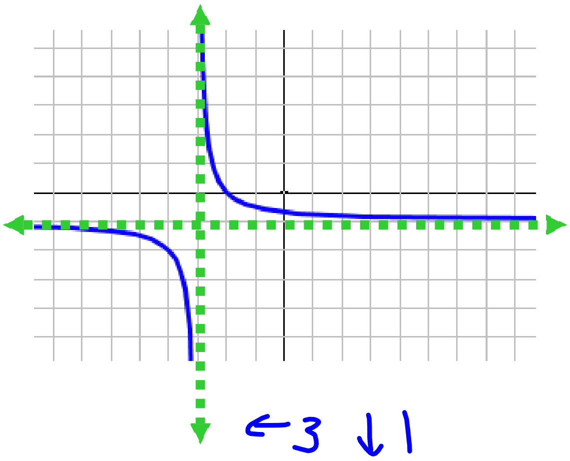
Decreasing:

End Behavior:

Asymptote behavior:

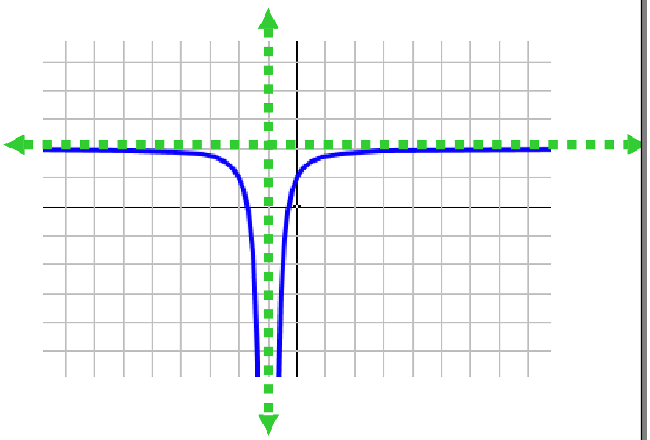


Based on the conclusions you made, work with a partner to write an equation based on the following graphs.



$\leftarrow 3 \quad \downarrow 1$

$$\frac{1}{(x+3)} - 1$$



$$-\frac{1}{(x+1)^2} + 2$$