

7-1 Defining and evaluating logarithms

- I can convert between exponential and logarithmic form
- I can evaluate logarithms

Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.

Exponential Equation

Logarithmic Equation

$$\begin{array}{ccc} b^x = a & \longleftrightarrow & \log_b a = x \\ 3^2 = 9 & & \log_3 9 = 2 \\ & & b > 0, b \neq 1 \end{array}$$

Examples

Exponential Equation	Logarithmic Equation
$4^3 = 64$	$\log_4 64 = 3$
$5^{-2} = \frac{1}{25}$	$\log_5 \frac{1}{25} = -2$
$\left(\frac{2}{3}\right)^p = q$	$\log_{\frac{2}{3}} q = p$
$\left(\frac{1}{2}\right)^n = m$	$\log_{\frac{1}{2}} m = n$

Switch between Log and exponential forms

Exponential Equation	Logarithmic Equation
$3^5 = 243$	$\log_3 243 = 5$
$4^{-3} = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$
$\left(\frac{3}{4}\right)^t = s$	$\log_{\frac{3}{4}} s = t$
$\left(\frac{1}{5}\right)^w = v$	$\log_{\frac{1}{5}} v = w$

The natural logarithm:

$$y = \ln x \text{ is equivalent to } x = e^y$$

logarithm natural

$$\ln x = y \iff e^y = x$$

$$\log_e x = y$$

$$\ln x = y$$

The common logarithm:

$$y = \log x \text{ is equivalent to } x = 10^y$$

Exponential Equation

$$e^5 \approx 148.4$$

$$e^{1.8} \approx 6$$

$$10^5 = 100,000$$

$$10^3 = 1,000$$

Logarithmic Equation

$$\ln 148.4 \approx 5$$

$$\ln 6 \approx 1.8$$

$$\log 100,000 = 5$$

$$\log 1,000 = 3$$

Evaluating logarithms

$$f(x) = \log x \quad \text{Find } f(10), f(0.1), f(100)$$

$$\log_{10} x =$$

$10^{\text{what exponent}} = x$

$$\star \log 10 = ? = 1 \Rightarrow 10^? = 10$$

$$\star \log 100 = 2 \Rightarrow 10^? = 100$$

$$\star \log 0.1 =$$

$$\log \left(\frac{1}{10}\right) = -1 \quad 10^? = \frac{1}{10}$$

Evaluate f(x)

$$f(x) = \log_2 x \quad \text{Find } f(4), f(16), f(64)$$

$$\log_2 4 = 2 \quad 2^? = 4$$

$$f(16) = \log_2 16 = 4 \quad 2^? = 16$$

$$f(64) = \log_2 64 = 6$$

Evaluate $f(x)$

$$f(x) = \log_7 x$$

Find $f(49)$, $f(343)$

$$\log_7 (49) = 2$$

Find the exact value

$$\log_2 32$$

$$\log_4 \frac{1}{16}$$

$$\log 10000000$$

$$\log .00001$$

Find the exact value

$$\log_5 25$$

$$\log_2 \frac{1}{8}$$

$$\log 1000$$

$$\log .001$$

Use a calculator to

First, find the common logarithm of 0.42. Round the result to the thousandths place and raise 10 to that number to confirm that the power is close to 0.42.

$$\log 0.42 \approx -0.38$$

$$10^{\square} \approx 0.42$$

Next, find the natural logarithm of 0.42. Round the result to the thousandths place and raise e to that number to confirm that the power is close to 0.42.

$$\ln 0.42 \approx -0.87$$

$$e^{\square} \approx 0.42$$

Your Turn

Use a scientific calculator to find the common logarithm and the natural logarithm of the given number. Verify each result by evaluating the appropriate exponential expression.

11. 0.25

common log:
 $\log 0.25 =$

natural log:
 $\ln (.25) =$

12. 4

$$\log 4 =$$

$$\ln 4 =$$