7-1 Rational Graphs

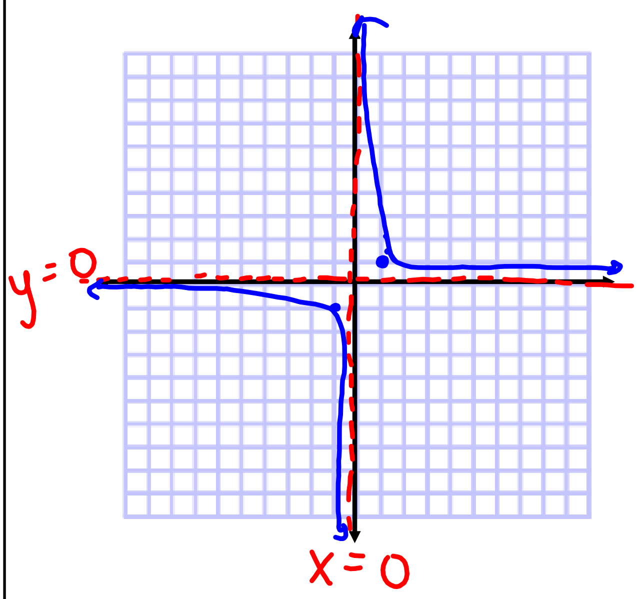
Objectives:

I can determine the domain, range, symmetry, end behavior, and intervals of increasing and decreasing of rational graphs.

I can identify the transformation of a given function and sketch a graph

I can write a rational equation given a graph.

$$f(x) = \frac{1}{x}$$



Domain $(-\infty, 0) \cup (0, \infty)$

Range $(-\infty, 0) \cup (0, \infty)$

Increasing None

Decreasing $(-\infty, 0) \cup (0, \infty)$

Left End Behavior

$\lim_{x \to -\infty} f(x) = 0$

Right End Behavior

$\lim_{x \to +\infty} f(x) = 0$

x-intercepts

y-intercepts NONE

Vertical Asymptote(s): $x = 0$

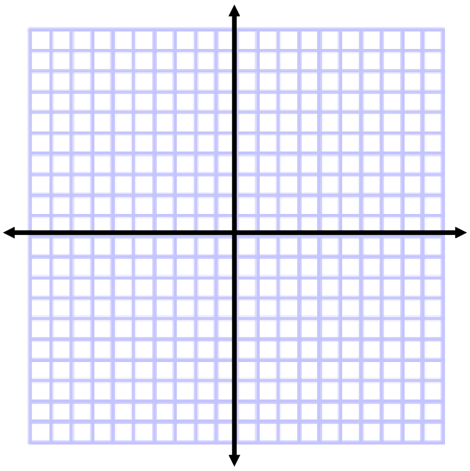
Horizontal Asymptote: $y = 0$

One-to-One?

Yes

Rational w/ odd power

Equation: $\frac{1}{x^3}$ or $\frac{1}{x^5}$



Domain

Range

Increasing

Decreasing

Left End Behavior

Right End Behavior

x-intercepts

y-intercepts

Vertical Asymptote(s):

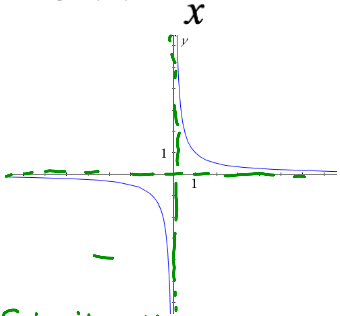
Horizontal Asymptote:

One-to-One?

looks the same as $1/x$

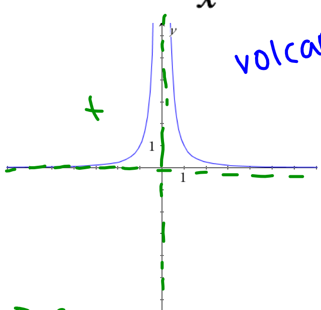
Look at the following Graphs $f(x)=\frac{1}{x}$ and $f(x)=\frac{1}{x^2}$ and compare. What is going on?

$$f(x)=\frac{1}{x}$$



$$f(x)=\frac{1}{x^2}$$

volcano



Similarities:

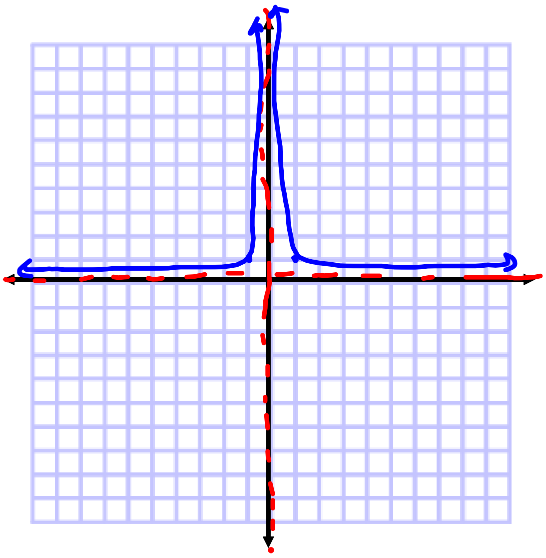
- Same asymptotes
- Top Right corners both decreasing / both positive
- Both ends going to zero
- Same domain

Differences:

- Left sides are opposite
- $1/x^2$ is a little steeper
- Ranges

Rational w/even power

Equation: $1/x^2$



Domain $(-\infty,0)\cup(0,\infty)$

Range $(0,\infty)$

Increasing $(-\infty,0)$

Decreasing $(0,\infty)$

Left End Behavior

$\lim_{x\to-\infty} f(x) = 0$

Right End Behavior

$\lim_{x\to\infty} f(x) = 0$

x-intercepts

y-intercepts NONE

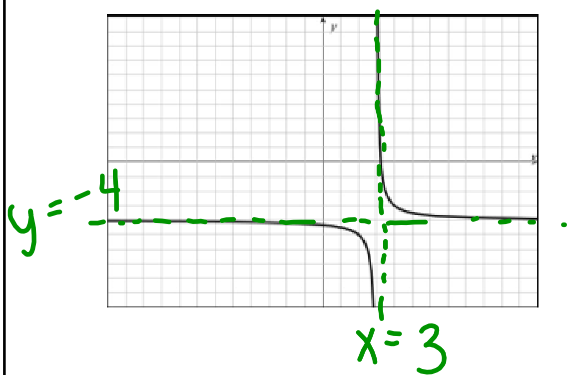
Vertical Asymptote(s): $x=0$

Horizontal Asymptote: $y=0$

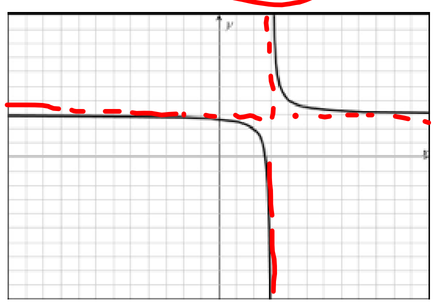
One-to-One?

NO

$$f(x)=\frac{1}{(x-3)}-4$$



$$f(x)=\frac{1}{x-3}+3$$



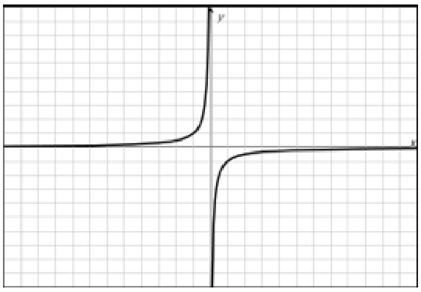
Based on the equations and corresponding graphs, what do you conclude about the transformations?

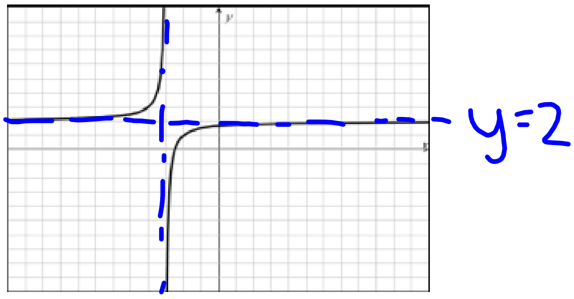
Shifted:
- Right 3
- Down 4

Shifted
- Right 3
- Up 3

$$f(x) = -\frac{1}{x}$$

$$f(x) = -\frac{1}{x+3} + 2$$





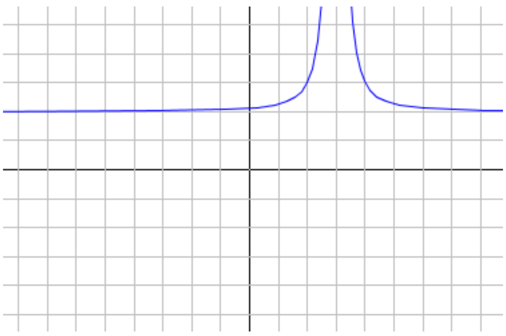
V. Flip

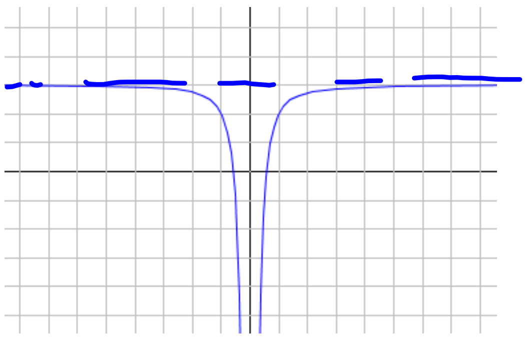
up 2
Left 3
V. Flip

Based on the equations and corresponding graphs, what do you conclude about the transformations?

$$f(x) = \frac{1}{(x-3)^2} + 2$$

$$f(x) = -\frac{1}{x^2} + 3$$





Based on the equations and corresponding graphs, what do you conclude about the transformations?

Sketch a graph and analyze of the following.

$$f(x) = -\frac{1}{x} + 3$$

Domain:

Range:

V Asymptote:

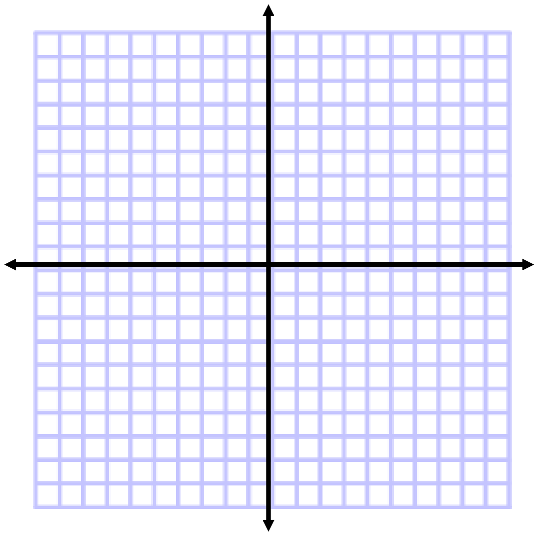
H Asymptote:

Increasing:

Decreasing:

End Behavior:

Asymptote behavior:



Sketch a graph and analyze of the following.

$$f(x) = \frac{1}{(x+3)^2} + 1$$

Domain:

Range:

V Asymptote:

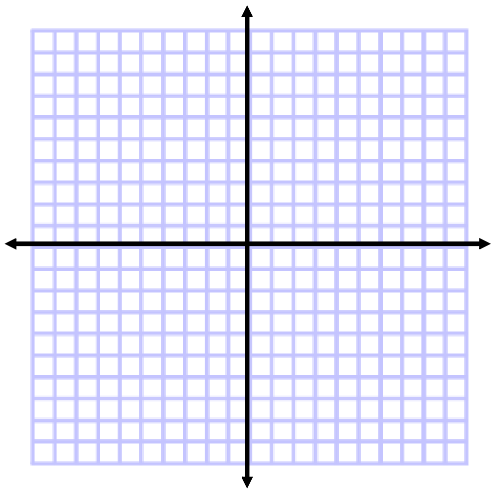
H Asymptote:

Increasing:

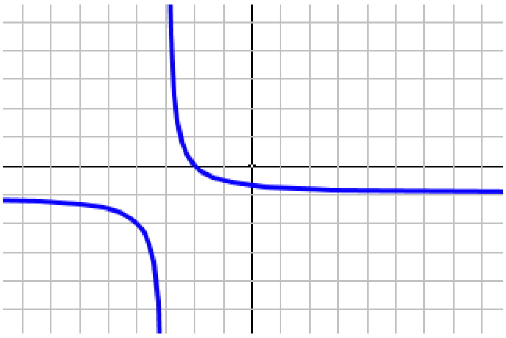
Decreasing:

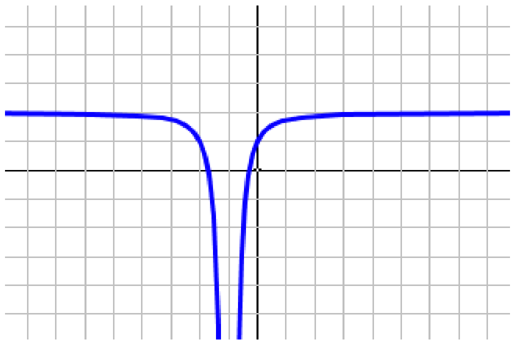
End Behavior:

Asymptote behavior:



Based on the conclusions you made, work with a partner to write an equation based on the following graphs.





When given a rational function in the form of $f(x)=\dfrac{mx+n}{px+q}$ where $m \neq 0$ and $p \neq 0$, you can use division to re-write the function in a form to identify the transformations.

$$g(x)=\frac{3x-4}{x-1}$$

$(3x-4)\div(x-1)$

1 | 3   -4

\+ ↓   3

3   -1

$g(x)=\dfrac{-1}{(x-1)}+3$

(x^3+5x^2-2x+1)

$\div(x-3)$

3 | 1   5   -2   1

\+ ↓   3   24   66

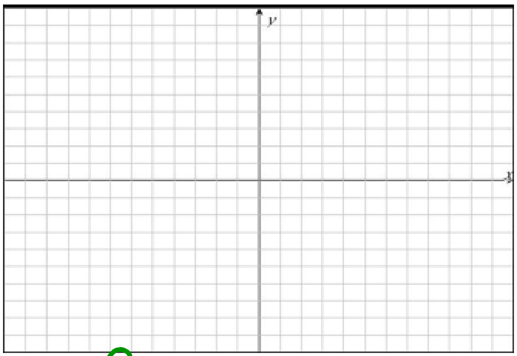
1   8   22 | 67

Given $f(x)=\dfrac{4x+7}{x+4}$, use division to re-write the function and identify the transformations. Then sketch a graph and state the domain, range, and intervals of increasing and decreasing.

$$\begin{array}{r|rr} -4 & 4 & 7 \\ + & \downarrow & -16 \\ \hline & 4 & -9 \end{array}$$

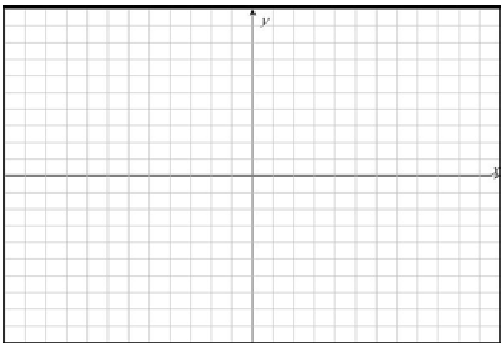
$$f(x)=\frac{-9}{(x+4)}+4$$

V. Flip
V. Stretch by 9



$\dfrac{R}{\text{denom.}}+\#$

Given $f(x)=\dfrac{3x+7}{x+2}$, use division to re-write the function and identify the transformations. Then sketch a graph and analyze.



$$f(x) = \frac{5-2x}{x+4}$$