7-2 Properties of Logarithms

I can use the properties of logarithms to simplify logarithms.

I can use the properties of logarithms to express logarithms in different ways.

$$\log_a 1 = \underline{\underline{0}} \qquad \log_a a = \underline{\underline{1}}$$

any base

match

Evaluate

$\log_5 1 = 0$

$5^? = 1$

$\ln 1 = 0$

$\log_4 4 = 1$

$4^? = 4$

$\log_{10} 10 = 1$

Inverse Property of Logarithms
If b and M are positive real numbers, with $b \neq 0$, then

$$b^{\log_b M} = M$$

match

Evaluate

$\cancel{5^{\log_5}}\,{}^{20} = 20$ $\qquad$ $\cancel{8^{\log_8}}\,{}^{12} = 12$

Evaluate

$\cancel{12^{\log_{12}}}\,{}^{3} = 3$ $\qquad$ $\cancel{10^{\log_{10}}}\,{}^{6}$

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Inverse Property of Logarithms

If b and r are positive real numbers, with $b \neq 0$, then

$$\log_a a^r = r$$

cancel

Evaluate

$\cancel{\log_4 4}^3 = ? = 3$

$4^? = 4^3$

$\cancel{\ln e}^7 = 7$

Evaluate

$\cancel{\log_8 8}^3$

3

$\cancel{\log 10}^{-4}$

-4

Product Rule of Logarithms

If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(MN) = \log_b M + \log_b N$$

Write each of the following logarithms as the sum of logarithms.

$\log_2(5 \cdot 3) = \log_2 5 + \log_2 3$

$\ln(6z) = \ln(6) + \ln(z)$

$\log_2(4x) = \underbrace{\log_2 4}_{2} + \log_2 x$

$2 + \log_2 x$

Write as a sum of logarithms

$\log_4(9 \cdot 5) = \log_4(9) + \log_4(5)$

$\log(5w) = \log 5 + \log w$

Product Rule of Logarithms

If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_2\left(\frac{5}{3}\right) = \log_2 5 - \log_2 3$$

$$\log\left(\frac{y}{5}\right) = \log y - \log 5$$

Write as a difference of logarithms

$$\log_7\left(\frac{9}{5}\right) = \log_7 9 - \log_7 5$$

$$\ln\left(\frac{p}{3}\right) = \ln(p) - \ln(3)$$

Write the following as the sum or difference of logarithms.

$$\log_3\left(\frac{3m}{n}\right)$$

$\log_3 3 + \log_3 m - \log_3 n$

$$\log_3\left(\frac{q}{3p}\right)$$

$\log_3 q - \log_3 3$

$-\log_3 p$

Product Rule of Logarithms

If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r \cdot \log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$$\log_8 3^5$$

$5 \cdot \log_8 3$

$$\ln x^3$$

$3 \cdot \ln x$

Use the power Rule of Logarithms to express all powers as factors.

$\log_2 5^6$

$6 \cdot \log_2 5$

$\log b^5$

$5 \cdot \log b$

$5 \log(b)$

Expand the following logarithms.

$\log_2(x^2 \cdot y^3)$

$\log_2 x^2 + \log_2 y^3$

$\boxed{2 \log_2 x + 3 \log_2 y}$

$\log\left(\frac{100 \cdot x}{y}\right)$

$\log 100 + \log x - \log y$

Expand the following logarithms.

$\log_4(a^2b) = \log_4 a^2 + \log_4 b$

$\boxed{2 \cdot \log_4 a + \log_4 b}$

$\log_3\left(\frac{9m^4}{n}\right) =$

$\log_3 9 + \log_3 m^4 - \log_3 n$

$\log_3 9 + 4\log_3 m - \log_3 n$

Write each of the following as a single logarithm.

$\log_6 3 + \log_6 12 = \log_6(3 \cdot 12)$

$\log_6(36)$

$\log(x-2) - \log x = \log\left(\frac{x-2}{x}\right)$

Write each of the following as a single logarithm.

$$\log_8 4 + \log_8 16$$

$$\log_3(x+4) - \log_3(x-1)$$

Write each of the following as a single logarithm.

$$2\log_2(x-1) + 3\log_2 x$$

$$\log_2 (x-1)^2 + \log_2 x^3$$

$$\log_2\left((x-1)^2 x^3\right)$$

$$\log(x+1) - 4\log x$$

$$\log (x+1) - \log x^4$$

$$\boxed{\log\left(\frac{(x+1)}{x^4}\right)}$$

Change of Base Formula

If $a \neq 0$, $b \neq 0$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

which means:

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

Use your calculator to approximate the following:

$$\log_4 45 = \frac{\log(45)}{\log(4)}$$

$$\log_3 26 \quad \frac{\ln(26)}{\ln(3)}$$

Summary of Properties

$$\log_a a^r = r \qquad b^{\log_b M} = M$$

$$\log_b(MN) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b M^r = r\log_b M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$