7-2a: Asymptotes of Rational Functions

Objectives:

1. I can find the holes and vertical asymptotes of a rational function.
2. I can find the x- and y-intercepts of a rational function.
3. I can find the end behavior models of a rational function.
4. I can analyze a graph of a rational function.
5. I can graph a rational function by hand.

H. Asymptotes

Holes and Vertical asymptotes

$$f(x) = \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \frac{x+3}{x+1}$$

1. Simplify (what cancels?)

what cancels gives us a hole (x-2) cancelled

hole: x = 2

Any excluded value that doesn't cancel = Vertical Asymptote

(x+1) = 0 (set bottom = 0)

VA: x = -1

Find the holes and vertical asymptotes

a. $y = \dfrac{5x}{x+2} = 0$

Holes: None

VA: $x = -2$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$

b. $y = \dfrac{2x^3}{x-5}$

Holes: None

VA: $x = 5$

c. $y = \dfrac{\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} = \dfrac{1}{x-2}$

Holes: $x = -2$

VA: $x = 2$

d. $f(x) = \dfrac{x^2-9}{x^2-5x+6} = \dfrac{(x+3)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}$

Holes: $x = +3$

VA: $x = 2$

X and Y Intercepts

Y intercepts, x = 0

$(0, y)$

$f(x) = \dfrac{3x-12}{x^2-5x-6} = \dfrac{3(0)-12}{0^2-5(0)-6} = \dfrac{-12}{-6} = 2$

Plug 0 in for x

$\boxed{(0,2)}$

X intercepts, y = 0 (top = 0)

$(x, 0)$

$f(x) = \dfrac{3x-12}{x^2-5x-6} = 0$

$3x - 12 = 0$
$+12 \quad +12$
$\dfrac{\cancel{3}x}{\cancel{3}} = \dfrac{12}{3}$

$x = 4$

$\boxed{(4,0)}$

Find the x and y intercepts of the following functions:

$$f(x)=\frac{x^2-\overset{-1}{2}x-\overset{-3}{3}}{x+2}$$

$$f(x)=\frac{3x-5}{x^2-5x+6}$$

$$f(x)=\frac{(x-3)(x+1)}{(x+2)}$$

x-int: $(3,0),(-1,0)$

$(x-3)(x+1)=0$

y-int: $(0,-3/2)$

$$\frac{(0-3)(0+1)}{(0+2)}=\frac{(-3)(1)}{2}=\frac{-3}{2}$$

Horizontal Asymptotes
(End Behavior Models)

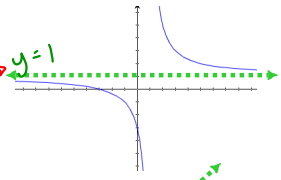
Look at the graphs, see if you can find the end behavior models. What are the patterns?

$f(x)=\frac{1x+3}{1x-1}$ deg: 1 / deg: 1

Equal

$y=\frac{1}{1}=1$

$y=1$

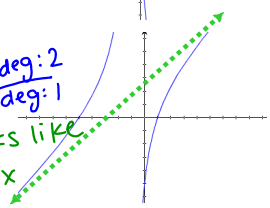


$f(x)=\frac{(x+5)(x-1)}{(x+1)}$ $\frac{\text{deg: 2}}{\text{deg: 1}}$

Top Heavy

y = slant asymptote

looks like $y=x$

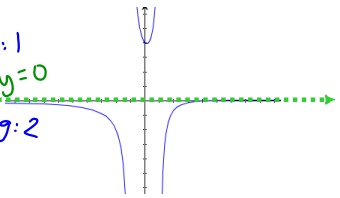


$f(x)=\frac{(x-4)}{(x+1)(x-1)}$ deg: 1 / deg: 2

$y=0$

Bottom Heavy:

$y=0$

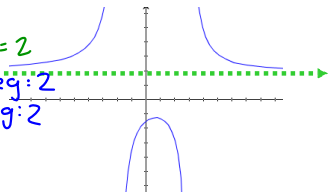


$f(x)=\frac{2x^2-3x+9}{1x^2-x-6}$ $\frac{\text{deg: 2}}{\text{deg: 2}}$

Equal

$y=\frac{2}{1}=2$

$y=2$



Horizontal Asymptotes (End Behavior):

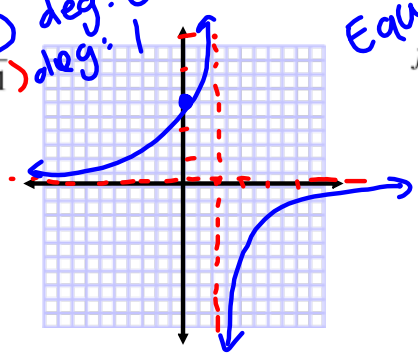
To find the Horizontal Asymptote (end behavior model), compare the degrees of the numerator and denominator.

Top heavy: y = slant asymptote
Bottom heavy: y = 0
Equal: y = divide leading coefficients

Identify the holes, vertical asymptotes, x and y intercepts, end behavior, and then make a sketch.

Bottom

$f(x) = \frac{-3}{(x-1)}$ deg: 0 deg: 1



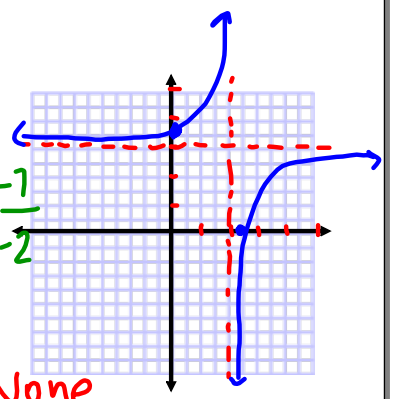
Holes: None
VA: $x = 1$
X-int: None
Y-int: $(0,3)$
HA: $y = 0$

$\frac{-3}{0-1} = \frac{-3}{-1}$

Equal

$f(x) = \frac{3x-7}{x-2}$

$\frac{3(0)-7}{0-2} = \frac{-7}{-2}$



Holes: None
VA: $x = 2$
X-int: $(7/3, 0)$
Y-int: $(0, 7/2)$
HA: $y = \frac{3}{1} = 3$

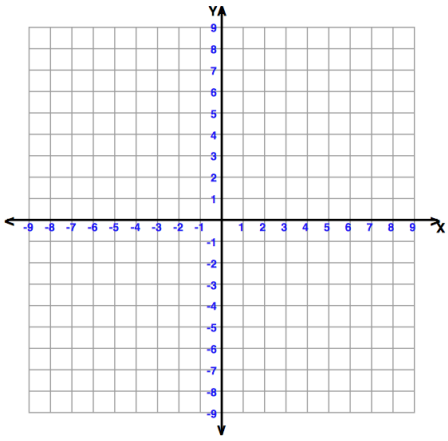
$3x - 7 = 0$
$+7 \quad +7$
$\frac{3x}{3} = \frac{7}{3}$
$x = \frac{7}{3}$

7-2b Graphing Rational Functions

Equal

$$f(x) = \frac{3x-2}{x-1}$$

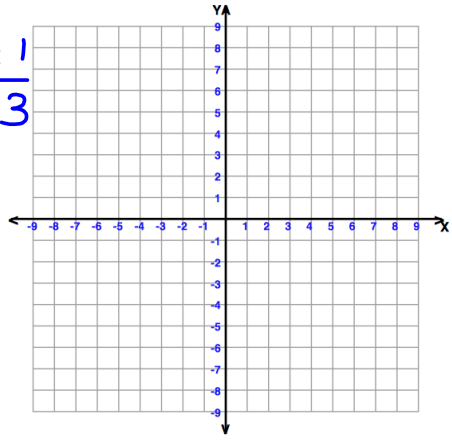
$\frac{\text{deg: 1}}{\text{deg: 1}}$



Bottom

$$f(x) = \frac{-(x+3)}{(x-1)^2(x+2)}$$

$\frac{\text{deg: 1}}{\text{deg: 3}}$



Non-Horizontal End Behavior

Top heavy rational functions havenon-horizontal end behaviors

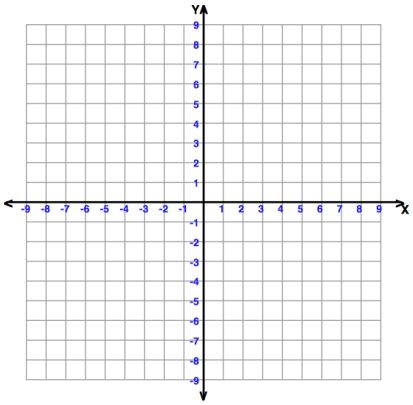
To find the degree of the end behavior**model (EBM)** - divide the leading terms and reduce.

the ends of $\dfrac{3x^5 - 4x^2 + 5}{2x^3 - 5x + 4}$ will behave like $\dfrac{3x^5}{2x^3} = \boxed{\dfrac{3x^2}{2}}$

Ex. 5 Find all asymptotes/EBM, holes and graph.

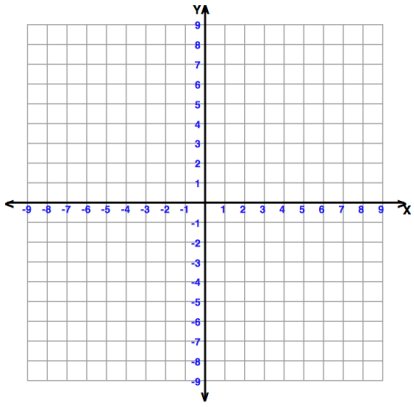
Top

a. $f(x) = \dfrac{x^3}{x^2 - 9}$



Equal

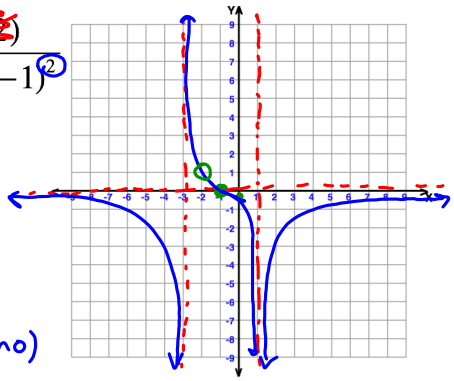
b. $f(x) = \dfrac{x^2 - 9}{x^2 - 5x + 6}$



c. $f(x) = \dfrac{-(x+1)(\cancel{x+2})}{(x+3)(\cancel{x+2})(x-1)^2}$

Bottom

deg: 2
deg: 4



Holes: $x = -2$

VA: $x = -3$ (opposite), $x = 1$ (volcano)

x-int: $(-1, 0)$

y-int: $(0, -1/3)$

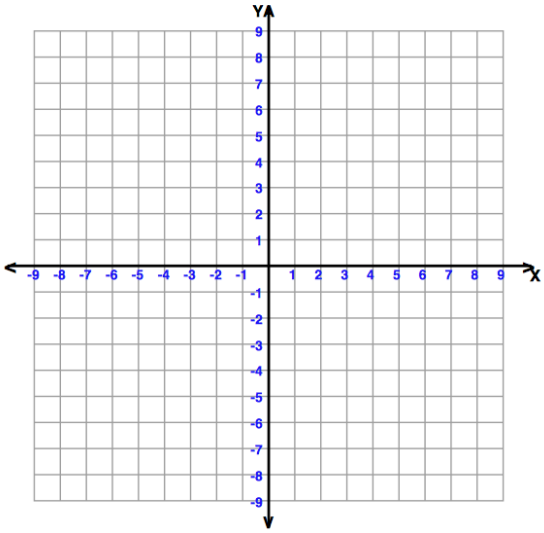
$\dfrac{-(1)}{(3)(-1)^2} = \dfrac{-1}{3}$.

HA: $y = 0$

Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

Bottom

$$f(x) = \frac{x^1 - 1}{x^2 - x - 12}$$



Domain
Range
x-intercepts
y-intercepts
VA
HA
Increasing
Decreasing
Continuous
Asymptote Behavior

End Behavior