

Asymptotes of Rational Functions

A4 & B8

State the domain using interval notation. For any x -value excluded from the domain, state whether the graph has a vertical asymptote or a "hole" at that x -value. Use a graphing calculator to check your answer.

1. $f(x) = \frac{x+5}{x+1}$ (x ≠ -1 = V. Asymptote)

Domain: $(-\infty, -1) \cup (-1, \infty)$

2. $f(x) = \frac{x^2 + 2x - 3}{x^2 - 4x + 3}$ } factor 1st!

Find any holes, asymptotes, and intercepts and state the end behavior.

3. $f(x) = \frac{x-1}{x^2+x-6} = \frac{(x-1)}{(x+3)(x-2)}$

Holes: None

VA: $x = -3, x = 2$

HA: $y = 0$

X-int: $(1, 0)$

Y-int: $(0, \frac{1}{4})$

$$\frac{(0-1)}{0^2+0-6} = \frac{-1}{-6} = \frac{1}{6}$$

EB:
 $\lim_{x \rightarrow -\infty} f(x) = 0$ $\lim_{x \rightarrow \infty} f(x) = 0$

7. $f(x) = \frac{x-1}{x+1}$

Sketch the graph of the given rational function. Also state the function's domain and range using interval notation. Find any x and y intercepts, state the end behavior, and behavior around the asymptotes.

8. $f(x) = \frac{x+1}{(x-1)^2(x+2)}$

Domain:

Range:

X-intercept:

Y-intercept:

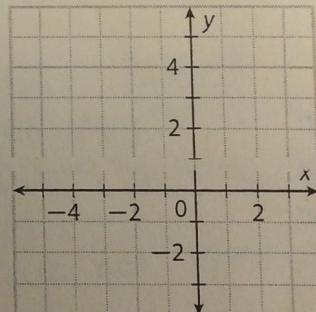
V Asymptote:

Hole:

increasing:

decreasing:

End Behavior:



Asymptotes Behavior:

$$9. f(x) = \frac{x^2 + 2x - 3}{x^2 + x - 2} = \frac{(x+3)(x-1)}{(x+2)(x-1)}$$

H. Asymptote: $y = 1$
 Domain: $(-\infty, -2) \cup (-2, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

X-intercept: $(-3, 0)$ $x+3=0 \quad x=-3$

Y-intercept: $(0, 3/2)$ $\frac{0+3}{0+2} = \frac{3}{2}$

V Asymptote: $x = -2$ $x+2=0 \quad x=-2$

Hole: $x = 1$ $x-1=0 \quad x=1$

increasing: None

decreasing: $(-\infty, -2) \cup (-2, \infty)$

End Behavior:

Left

$$\lim_{x \rightarrow -\infty} f(x) = 1$$

Right

$$\lim_{x \rightarrow \infty} f(x) = 1$$

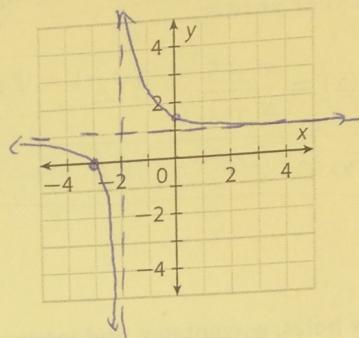
Asymptotes Behavior:

$$\lim_{x \rightarrow -2^-} f(x) = -\infty$$

from the left

$$\lim_{x \rightarrow -2^+} f(x) = +\infty$$

from the right



$$10. f(x) = \frac{-3x(x-2)}{(x-2)(x+2)}$$

Domain:

Range:

X-intercept:

Y-intercept:

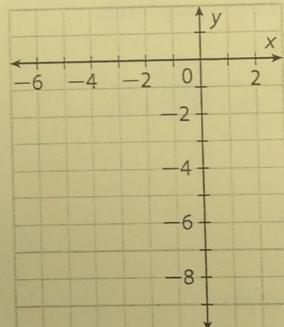
V Asymptote:

Hole:

Increasing:

Decreasing:

End Behavior:



Asymptotes Behavior:

11. $f(x) = \frac{x^2 - 1}{x + 2} = \frac{(x+1)(x-1)}{(x+2)}$

H.A.: Slant looks like $\frac{x^2}{x} = x$ (diagonal line)
 Domain: $(-\infty, -2) \cup (-2, \infty)$

Range:

X-intercept: $(-1, 0) (1, 0)$

Y-intercept: $(0, -\frac{1}{2})$

V Asymptote: $x = -2$

Hole: None

increasing:

decreasing:

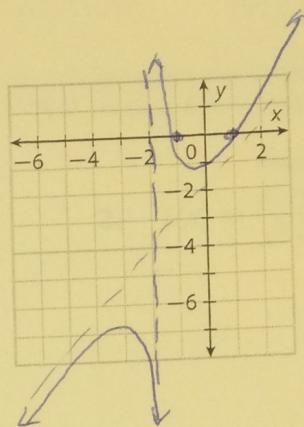
End Behavior:

$$\lim_{x \rightarrow -\infty} f(x) = -\infty \quad \lim_{x \rightarrow \infty} f(x) = +\infty$$

$$(x+1)(x-1) = 0$$

$$\frac{(x+1)(x-1)}{(x+2)} > \frac{-1}{2}$$

$$(x+2) = 0$$



Asymptotes Behavior:

$$\lim_{x \rightarrow -2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -2^+} f(x) = +\infty$$

18. Draw Conclusions For what value(s) of a does the graph of $f(x) = \frac{x+a}{x^2 + 4x + 3}$

have a "hole"? Explain. Then, for each value of a , state the domain and the range of $f(x)$ using interval notation.

19. Critique Reasoning A student claims that the functions $f(x) = \frac{4x^2 - 1}{4x + 2}$ and $g(x) = \frac{4x + 2}{4x^2 - 1}$ have different domains but identical ranges. Which part of the student's claim is correct, and which is false? Explain.

$$f(x) = \frac{(2x+1)(2x-1)}{2(2x+1)} \quad x \neq -\frac{1}{2}$$

$$D: (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$$

$$g(x) = \frac{2(2x+1)}{(2x+1)(2x-1)} \quad x \neq -\frac{1}{2}, \frac{1}{2}$$

$$D: (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

★ Different Domains is correct. Ranges are not identical.

Review

Simplify the following rational expressions

1. $\frac{2}{x^2 - x - 2} \cdot \frac{10}{x^2 + 2x - 8}$

2. $\frac{x}{x^2 - 6x + 8} \cdot \frac{1}{x^2 - x - 12}$