7-3: Graphing Rational Functions

Objectives:

1. I can find the x and y intercepts of a rational function
2. I can find the vertical and horizontal asymptotes of a rational function
3. I can find the holes of a rational function
4. I can analyze a graph of a rational function
5. I can graph a rational function by hand

X and Y Intercepts

Y intercepts, x = 0

(0, y)

$$f(x)=\frac{3x-12}{x^2-5x-6}$$

- Plug 0 in for x

$$\frac{3(0)-12}{0^2-5(0)-6}=\frac{-12}{-6}=2$$

(0, 2)

X intercepts, y = 0

(x, 0)

$$f(x)=\frac{3x-12}{x^2-5x-6}=0$$

set top equal to 0, solve for x

$3x-12=0$

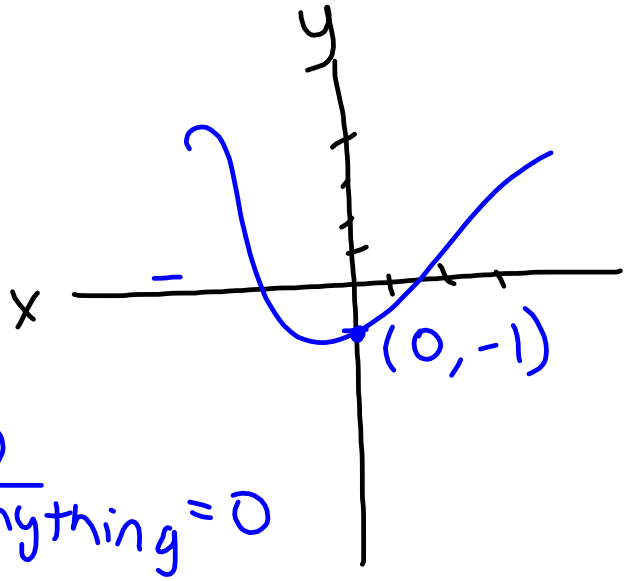
$+12 \quad +12$

$3x=12$

$\frac{3x}{3}=\frac{12}{3}$

$x=4$

(4, 0)



$\frac{0}{\text{anything}} = 0$

$\frac{3x-12}{x^2-5x+6} = 0$

Find the x and y intercepts of the following functions:

$$f(x) = \frac{(x-3)(x+1)}{x+2} = 0$$

x-int : $(-3, 0), (1, 0)$

y-int : $(0, -3/2)$

$\frac{(0-3)(0+1)}{(0+2)} = \frac{(-3)(1)}{(2)} = \frac{-3}{2}$

$$f(x) = \frac{3x-5}{(x-2)(x-3)} = 0$$

x-int: $(5/3, 0)$

$3x - 5 = 0$

$\frac{3x}{3} = \frac{5}{3}$

y-int: $(0, -5/6)$

$\frac{3(0)-5}{(0-2)(0-3)}$

$\frac{-5}{(-2)(-3)} = \frac{-5}{6}$

Review of Vertical Asymptotes

$$f(x)=\frac{2}{x+3}$$

$$\frac{5}{(x+7)(x-2)}$$

$x=-7, 2$

Set the denominator = 0, then solve for x

$x+3=0$

$-3\ \ -3$

$x=-3$

Find the vertical asymptotes:

a. $y=\dfrac{3x-5}{(x-2)(x+2)}$

VA: $x=2, -2$

b. $y=\dfrac{2x^3}{x-5}$

VA: $x=5$

c. $y=\dfrac{5x}{x+2}$

VA

$x=-2$

Asymptotes:

check for holes before VA!! (by reducing the fraction if possible)

$$f(x)=\frac{(x-3)(x-2)}{(x-2)} = x-3$$

since (x-2) cancels, Hole: $x=2$

vertical (VA): caused by dividing by 0

the graph approaches $-\infty$ or ∞

on each side of the asymptote

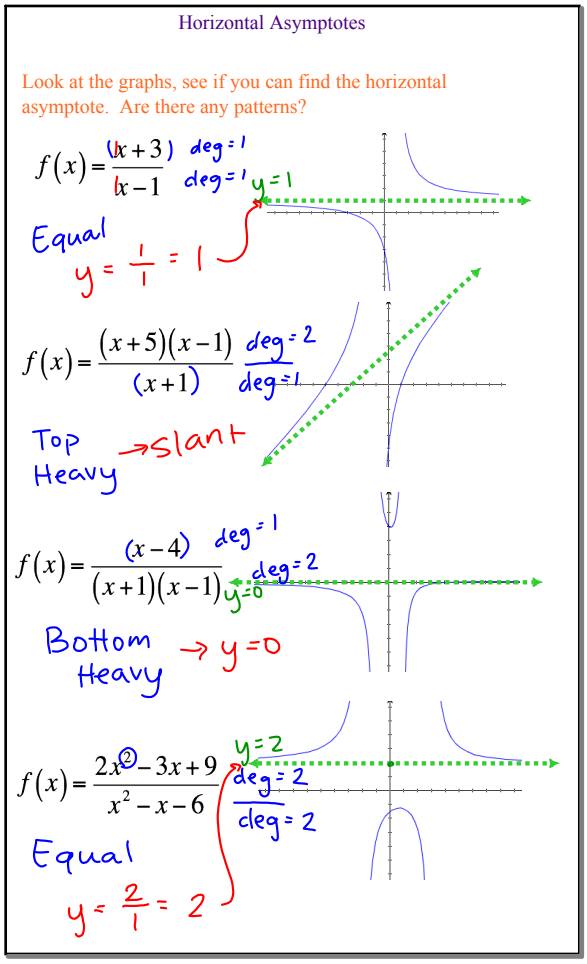
to find the asymptote set den = 0 and solve

Identify any holes, then find all vertical asymptotes

$$f(x)=\frac{(x-3)(x+3)}{(x-2)(x-3)} = \frac{x+3}{x-2},$$

Holes: $x=3$

VA.: $x=2$



end behavior: (horizontal (HA) or ~~oblique~~ slant (OA)):

to find the asymptote - compare the degrees of the numerator and denominator if:

top heavy (~~OA~~): slant asymptote
bottom heavy (~~HEB~~): y = 0
equal (~~HA~~): divide coefficients

↑ the # in front of x's

Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

$f(x) = \frac{-3}{(x-1)}$ $= 0$ deg: 0 deg: 1

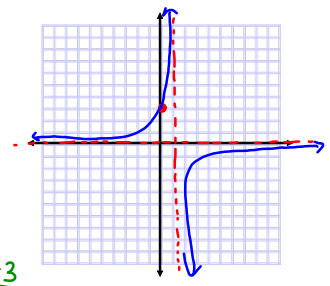
Bottom → HA: $y = 0$

Holes: None

VA: $x = 1$

x-int: None

y-int: $(0, 3)$ $\frac{-3}{0-1} = \frac{-3}{-1}$



$f(x) = \frac{3x-7}{(x-2)}$ deg: 1 deg: 1

Equal: HA $y = \frac{3}{1} = 3$

Holes: None

VA: $x = 2$

x-int: $(7/3, 0)$

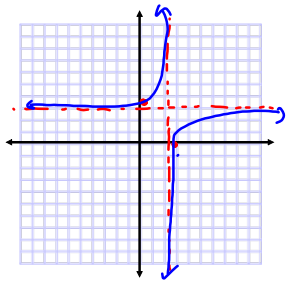
$3x - 7 = 0$

$+7 \quad +7$

$\frac{3x}{3} = \frac{7}{3}$ $x = \frac{7}{3}$

y-int: $(0, 3.5)$

$\frac{3(0)-7}{0-2} = \frac{-7}{-2} = +3.5 = 7/2$



Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

$f(x) = \frac{3x-2}{(x-1)}$

Equal: HA: $y = \frac{3}{1} = 3$

Holes: None

VA: $x = 1$

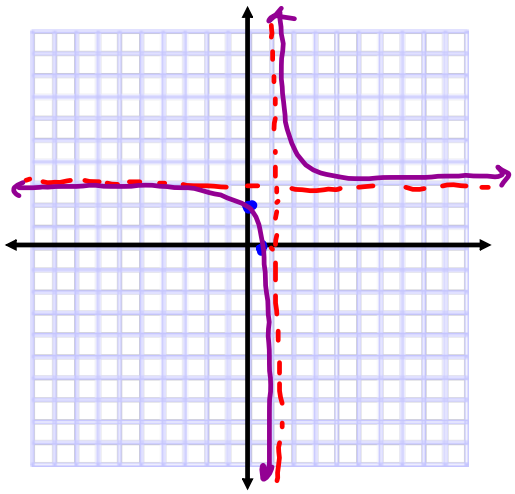
x-int: $(2/3, 0)$ $3x - 2 = 0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

y-int: $(0, 2)$

$\frac{3(0)-2}{(0-1)} = \frac{-2}{-1} = 2$



Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x)=\frac{x-1}{(x-4)(x+3)} \quad =0$$

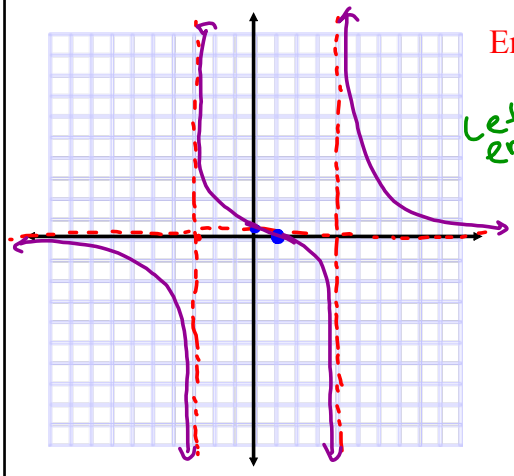
x
y

Holes: None
Domain $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$
Range $(-\infty,\infty)$
x-intercepts $(1,0)$
y-intercepts $(0,1/12)$
VA $x=4, x=-3$
HA $y=0$ (x-axis)
~~Asymptote Behavior~~

Bottom Heavy

$\frac{0-1}{(0-4)(0+3)}$

$= \frac{-1}{(-4)(3)} = \frac{-1}{-12} = \frac{1}{12}$



End Behavior

Left end: $\lim_{x\to-\infty} f(x)=0$

Right end: $\lim_{x\to\infty} f(x)=0$

H.A.

Graph and analyze $f(x)=\frac{4x+7}{1x+4} \quad =0$

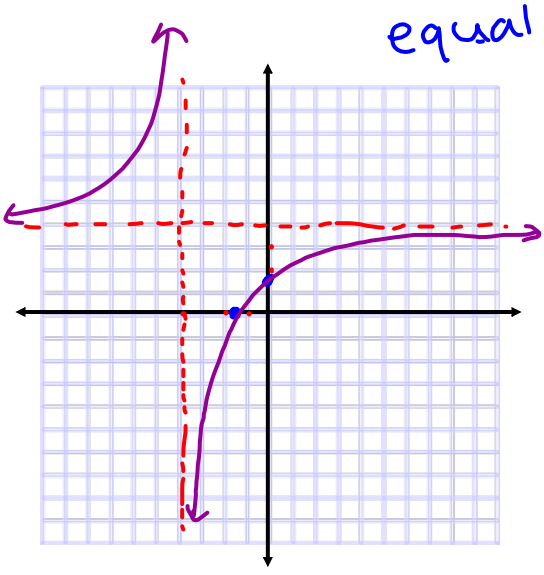
$4x+7=0$

$\frac{4x}{4}=\frac{-7}{4}$

$x=-7/4$

$\frac{7}{4}$

Domain $(-\infty,-4)\cup(-4,\infty)$
Range $(-\infty,4)\cup(4,\infty)$ 1.75
x-intercepts $(-7/4,0)$
y-intercepts $(0,7/4)$
VA $x=-4$
HA $y=\frac{4}{1}=4$ equal
~~Asymptote Behavior~~

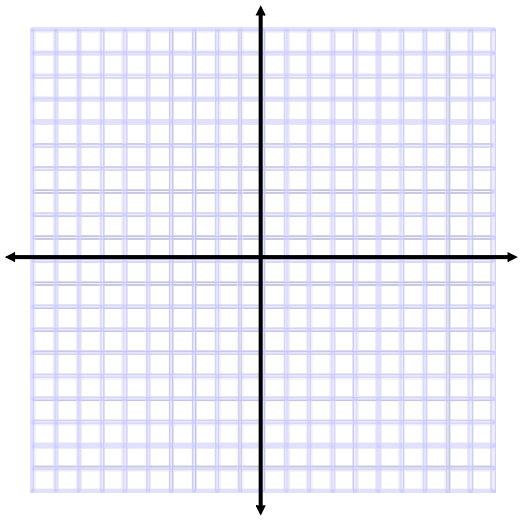


End Behavior

$\lim_{x\to\infty} f(x)=4$

$\lim_{x\to\infty} f(x)=4$

Graph and Analyze $f(x)=\dfrac{x+1}{(x+3)(x-4)}$

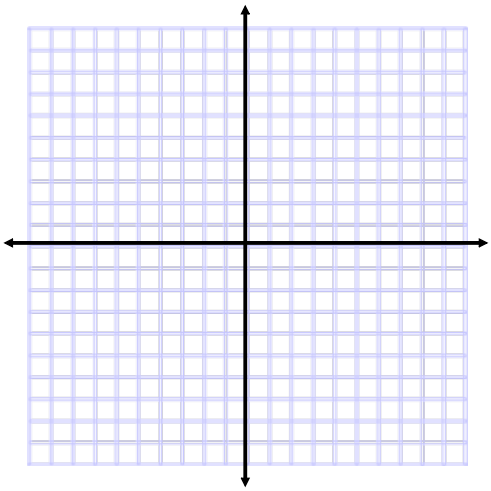


Domain
Range
x-intercepts
y-intercepts
VA
HA $y=0$

Asymptote Behavior

End Behavior

Graph and analyze $f(x)=\dfrac{4x-4}{x+2}$



Domain
Range
x-intercepts
y-intercepts
VA
HA

Asymptote Behavior

End Behavior