

## 8-1 Sequences

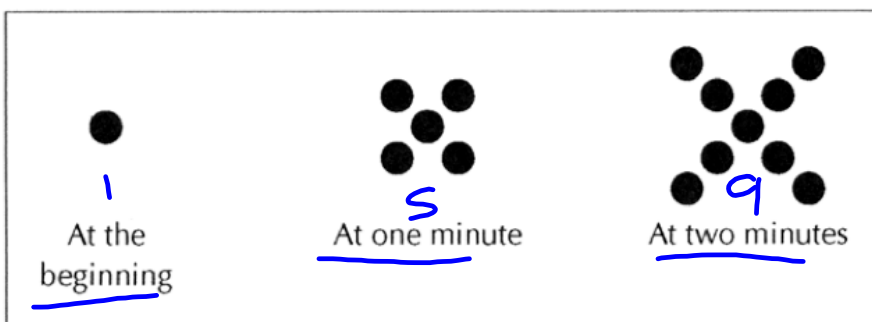
Objectives: I can write arithmetic and geometric sequences using explicit and recursive forms.

Write the next 3 terms for the following:

a.)  $\{5, 10, 20, 40, \underline{80}, \underline{160}, \underline{320}\}$

b.  $\{13, 21, 29, 37, \underline{45}, \underline{53}, \underline{61}\}$

c.  $\{15, 5, \frac{5}{3}, \frac{5}{9}, \underline{\frac{5}{27}}, \underline{\frac{5}{81}}, \underline{\frac{5}{243}}\}$



1. Describe the pattern that you see in the sequence of figures above.

gains 4 dots every minute

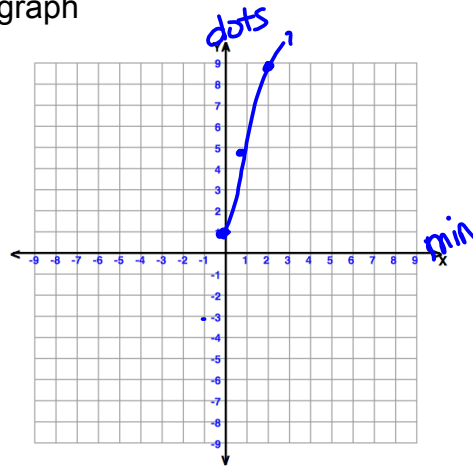
2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes? At 4 minutes?

3 minutes: 13 dots

4 minutes: 17 dots

3. Make a table of values and graph

| x min | y dots |
|-------|--------|
| 0     | 1      |
| 1     | 5      |
| 2     | 9      |
| 3     | 13     |



4. Write an equation to represent the pattern

~~$y = x + 4$~~

$$x = 4n + 1$$

↑
↑  
 dots            minutes

$$y = 4x + 1$$

↑
↑  
 dots            minutes

### Arithmetic Sequence

**arithmetic** - sequence with common difference between successive terms (repeated addition)

**explicit** - each term is defined independently

$$f(n) = a + dn \quad \text{for } n \geq 0$$

$1 + 4n$

**recursive** - use the previous term to define the following terms

$$f(0) = a, \quad f(n) = f(n-1) + d \quad n \geq 1$$

$$f(0) = 1, \quad f(n) = f(n-1) + 4$$

a = initial value (0<sup>th</sup> term)

d = common difference (# adding)

n = term number

**Example 1** Use the given table to write an explicit and a recursive rule for the sequence.

(A)

|        |   |   |   |    |    |    |
|--------|---|---|---|----|----|----|
| $n$    | 0 | 1 | 2 | 3  | 4  | 5  |
| $f(n)$ | 2 | 5 | 8 | 11 | 14 | 17 |

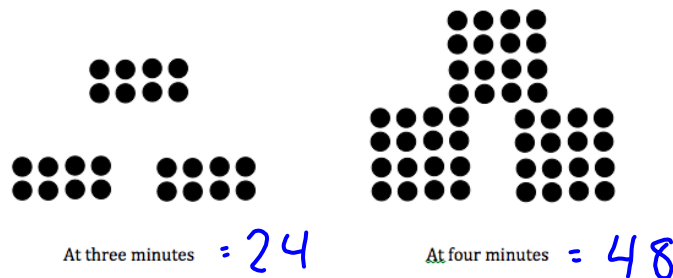
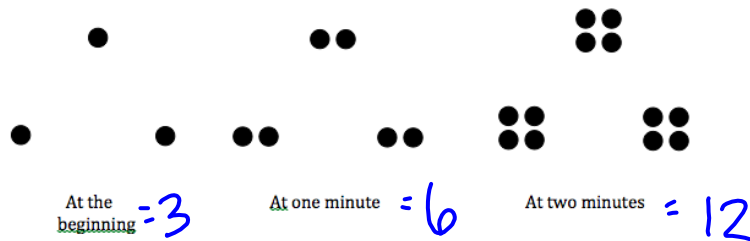
$a = 2$

$d = 3$

Explicit:  $f(n) = 3n + 2$

Recursive:  $f(0) = 2$

$f(n) = f(n-1) + 3$



1. Describe the pattern that you see in the sequence of figures above.

Multiplying by 2

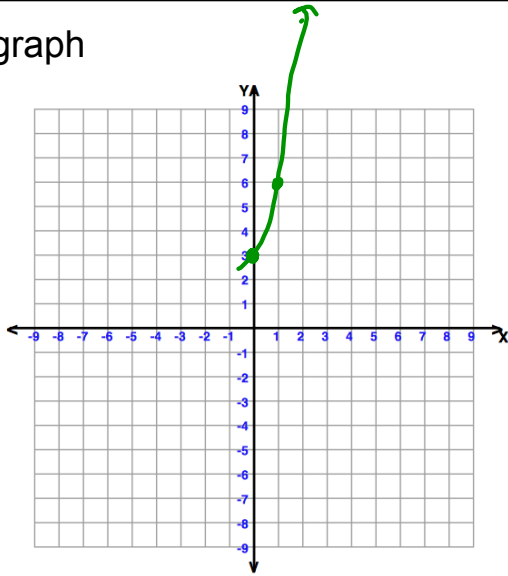
2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?

96 dots

II

3. Make a table of values and graph

| x min | y dots |
|-------|--------|
| 0     | 3      |
| 1     | 6      |
| 2     | 12     |
| 3     | 24     |



4. Write an equation to represent the pattern

Explicit:  
 $f(n) = 3(2)^n$

Recursive:  
 $f(0) = 3$   
 $f(n) = f(n-1) \cdot 2$

### Geometric Sequence

**geometric** - sequence with a common factor between successive terms (repeated multiplication)

ratio  
 ↙ # multiplying by

explicit:  $f(n) = a \cdot r^n$

recursive:  $f(n) = f(n-1) \cdot r$   
 $f(0) = a$

$a$  = initial value (0<sup>th</sup> term)

$r$  = common ratio (# multiplying by)

$n$  = term number

Write explicit and recursive rules to represent the table

(A)

|        |   |   |    |    |    |     |              |        |     |
|--------|---|---|----|----|----|-----|--------------|--------|-----|
| $n$    | 0 | 1 | 2  | 3  | 4  | ... | $j-1$        | $j$    | ... |
| $f(n)$ | 3 | 6 | 12 | 24 | 48 | ... | $ar^{(j-1)}$ | $ar^j$ | ... |

$\swarrow \searrow$   
 $\div 2 \quad \times 2 \quad \times 2 \quad \times 2$

$a = 3$

$r = 2$

Explicit:

$f(n) = 3(2)^n$

Recursive:

$f(0) = 3$

$f(n) = f(n-1) \cdot 2$

Write explicit and recursive rules to represent the table

(B)

|        |                 |                |               |   |   |    |     |              |        |     |
|--------|-----------------|----------------|---------------|---|---|----|-----|--------------|--------|-----|
| $n$    | 0               | 1              | 2             | 3 | 4 | 5  | ... | $j-1$        | $j$    | ... |
| $f(n)$ | $\frac{1}{125}$ | $\frac{1}{25}$ | $\frac{1}{5}$ | 1 | 5 | 25 | ... | $ar^{(j-1)}$ | $ar^j$ | ... |

$\swarrow \searrow \swarrow \searrow$   
 $\times 5 \quad \times 5 \quad \times 5$

$a = \frac{1}{125}$

$r = 5$

Geometric = Multiply

Explicit:  $f(n) = \frac{1}{125} (5)^n$

Recursive:

$f(0) = \frac{1}{125}$

$f(n) = f(n-1) \cdot 5$

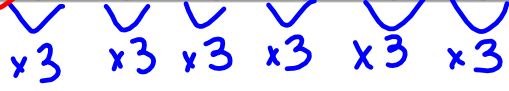
**Your Turn**

Write the explicit and recursive rules for a geometric sequence given a table of values.

4.

|        |                |               |               |   |   |   |    |     |
|--------|----------------|---------------|---------------|---|---|---|----|-----|
| $n$    | 0              | 1             | 2             | 3 | 4 | 5 | 6  | ... |
| $f(n)$ | $\frac{1}{27}$ | $\frac{1}{9}$ | $\frac{1}{3}$ | 1 | 3 | 9 | 27 | ... |

$a = \frac{1}{27}$   
 $r = 3$



Explicit:  $\frac{1}{27}(3)^n$

Recursive:  
 $f(0) = \frac{1}{27}$

$f(n) = f(n-1) \cdot 3$

5.

|        |       |      |     |   |    |     |      |     |
|--------|-------|------|-----|---|----|-----|------|-----|
| $n$    | 1     | 2    | 3   | 4 | 5  | 6   | 7    | ... |
| $f(n)$ | 0.001 | 0.01 | 0.1 | 1 | 10 | 100 | 1000 | ... |

**Example 3**

Write both an explicit and recursive rule for the geometric sequence that models the situation. Use the sequence to answer the question asked about the situation.

- (A) The Wimbledon Ladies' Singles Championship begins with 128 players. Each match, two players play and only one moves to the next round. The players compete until there is one winner. How many rounds must the winner play?



**Analyze Information**

Identify the important information:

- The first round requires \_\_\_\_\_ matches, so  $a =$  .
- The next round requires half as many matches, so  $r =$  .