You may pick your seat today, but no back row!

8-1 Sequences

Objectives: I can write arithmetic and geometric sequences using explicit and recursive forms.

Write the next 3 terms for the following:

a.) {5, 10, 20, 40, 80, 160, 320}

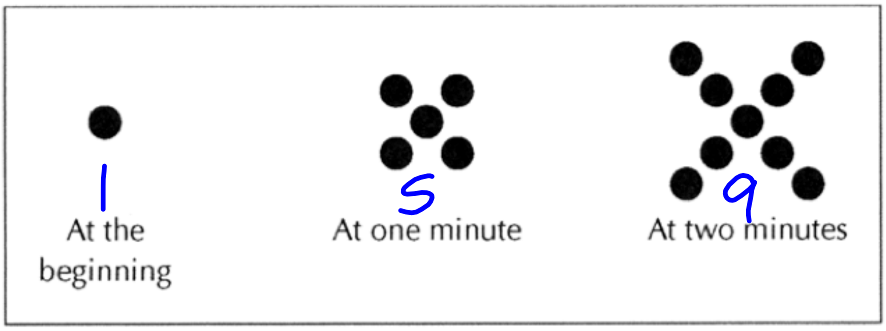
$\times 2$ $\times 2$ $\times 2$

b. {13, 21, 29, 37, 45, 53, 61}

$+8$ $+8$ $+8$

c. $\{15, 5, \frac{5}{3}, \frac{5}{9}, \frac{5}{27}, \frac{5}{81}, \frac{5}{243}\}$

$\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$



1. Describe the pattern that you see in the sequence of figures above.

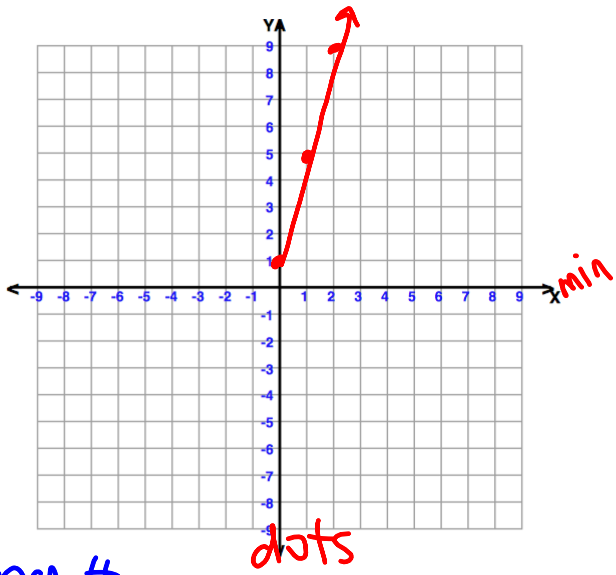
Adding 4 dots every minute

2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes? At 4 minutes?

13 at 3 min
17 at 4 min

3. Make a table of values and graph

x min	y dots
0	1
1	5
2	9
3	13



starting #

4. Write an equation to represent the pattern

$y = 1 + 4x$

adding

dots

min

Arithmetic Sequence

arithmetic - sequence with common differencebetween successive terms (repeated addition)

explicit - each term is defined independently

$$f(n) = a + dn \quad for \quad n \geq 0$$

recursive - use the previous term to define thefollowing terms

$$f(0) = a, \; f(n) = f(n-1) + d \quad n \geq 1$$

$f(0) = 1, \; f(n) = f(n-1) + 4$

a= initial value (0^{th} term)

d= common difference (# added)

n= term number

Example 1 Use the given table to write an explicit and a recursive rule for the sequence.

(A)

n	0	1	2	3	4	5	6
$f(n)$	2	5	8	11	14	17	20

+3 +3 +3

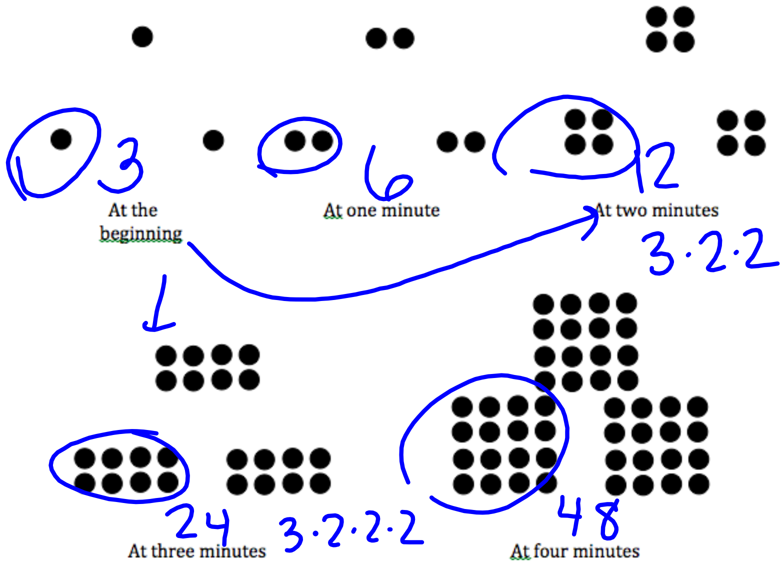
$a = 2$

$d = 3$

Explicit: $f(n) = 3 \cdot n + 2$

Recursive: $f(0) = 2$

$f(n) = f(n-1) + 3$



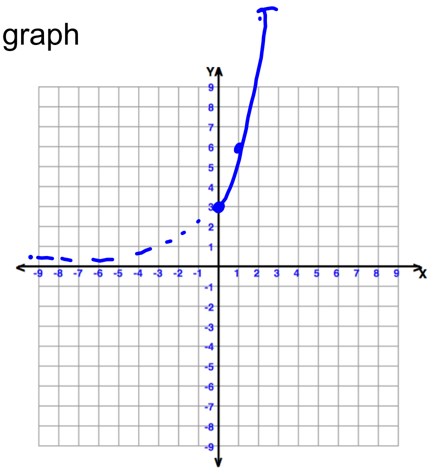
1. Describe the pattern that you see in the sequence of figures above.

Multiplying by 2

2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes? $48 \times 2 = 96$

3. Make a table of values and graph

x	y
0	3
1	6
2	12
3	24



4. Write an equation to represent the pattern

Explicit:

$\cancel{y = 2x + 3}$

Recursive:

$f(0) = 3$

$f(n) = f(n-1) \cdot 2$

(x) n = minutes

(y) f(n) = dots

Geometric Sequence

geometric - sequence with a common factor between successive terms (repeated multiplication)

explicit: $f(n) = a \cdot (r)^n$

recursive: $f(n) = f(n-1) \cdot r$

$f(0) = a$

a= 0th term (initial value)

r= common ratio (# multiplying)

n= term number

Write explicit and recursive rules to represent the table

(A)

n	0	1	2	3	4	...	$j-1$	j	...
$f(n)$	3	6	12	24	48	...	$ar^{(j-1)}$	ar^j	...

×2 ×2 ×2

$a = 3$

$r = 2$

Explicit: $f(n) = 3(2)^n$

Recursive: $f(0) = 3$

$f(n) = 2 \cdot f(n-1)$

Write explicit and recursive rules to represent the table

B

n	0	1	2	3	4	5	...	$j-1$	j	...
$f(n)$	$\frac{1}{125}$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	...	$ar^{(j-1)}$	ar^{j}	...

$\div 5$ $\times 5$ $\times 5$ $\times 5$ $\times 5$

$r = 5$

$a = \frac{1}{125}$

Explicit: $f(n) = \frac{1}{125}(5)^n$

Recursive: $f(0) = \frac{1}{125}$

$f(n) = 5 \cdot f(n-1)$

Your Turn

Write the explicit and recursive rules for a geometric sequence given a table of values.

4.

n	0	1	2	3	4	5	6	...
$f(n)$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	...

5.

n	1	2	3	4	5	6	7	...
$f(n)$	0.001	0.01	0.1	1	10	100	1000	...

Example 3 **Write both an explicit and recursive rule for the geometric sequence that models the situation. Use the sequence to answer the question asked about the situation.**

(A) The Wimbledon Ladies' Singles Championship begins with 128 players. Each match, two players play and only one moves to the next round. The players compete until there is one winner. How many rounds must the winner play?

Analyze Information

Identify the important information:

- The first round requires ______ matches, so $a =$ ☐.
- The next round requires half as many matches, so $r =$ ☐.

rounds	# games
0	128 players
1	64
2	32
3	16
4	

$\times\frac{1}{2}$

3, 15, 45, ...

1st term

$\frac{15}{3} = 5$

10, 5, 2.5, ...

$\frac{5}{10} = \frac{1}{2}$