8-4 Solving Radical Equations

Objectives:

1. I can solve radical equations and check for extraneous solutions.
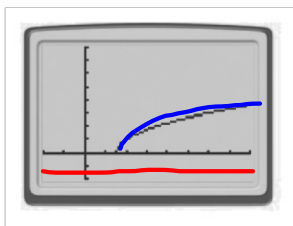
Mar 12-4:34 PM

Remember that you can graph the two sides of an equation as separate functions to find solutions of the equation: a solution is any x-value where the two graphs intersect.

The graph of $y = \sqrt{x-3}$ is shown on a calculator window of $-4 \le x \le 16$ and $-2 \le y \le 8$. Reproduce the graph on your calculator. Then add the graph of $y = 2$.

$$y = \sqrt{x-3}$$

$$y = 2$$

x-min: -4
x-max: 16
y-min: -2
y-max: 8

How many solutions does the equation $\sqrt{x-3} = 2$ have? ___1___ How do you know?

On your calculator, replace the graph of $y = 2$ with the graph of $y = -1$.

$y = -1$ no

How many solutions does the equation $\sqrt{x-3} = -1$ have? ___0___ How do you know?

Mar 12-4:50 PM

Graph both sides of $\sqrt{4x-4} = x+1$ as separate functions on your calculator.
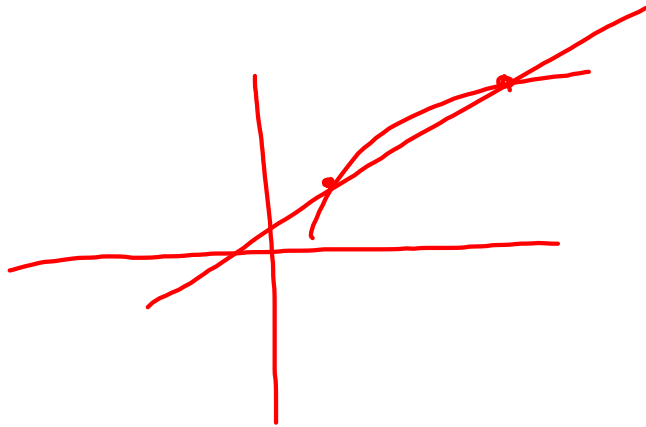
How many solutions does $\sqrt{4x-4} = x+1$ have? _0_

Replace the graph of $y = x+1$ with the graph of $y = \frac{1}{2}x$.

How many solutions does $\sqrt{4x-4} = \frac{1}{2}x$ have? _2_

$\frac{1}{2}X$

Replace the graph of $y = \frac{1}{2}x$ with the graph of $y = 2x-5$.

How many solutions does $\sqrt{4x-4} = 2x-5$ have? _____

Mar 12-4:53 PM

Solving Analytically

Solve.

$$4\sqrt{x} - 6 = 6$$

$\boxed{X=9}$

$$4\sqrt{x} - 6 + 6 = 6 + 6$$
$$4\sqrt{x} = 12 \qquad \text{Simplify.}$$

$4\sqrt{9} - 6 = 6$

Now divide each side by 4 to isolate the radical.

$$4\sqrt{x} = 12$$
$$\sqrt{x} = 3 \qquad \text{Divide and simplify.}$$

$4(3) - 6 = 6$

Next, square each side of the equation to eliminate the radical.

$$(\sqrt{x})^2 = 3^2$$
$$x = 9 \qquad \text{Simplify.}$$

$12 - 6 = 6$ ✓

Finally, check $x = 9$ in the original equation to verify that it is a solution and not an extraneous solution.

Mar 19-10:09 PM

2

Example 1   Solve the equation. Check for extraneous solutions.

Ⓐ $2 + \sqrt{x + 10} = x$

$-2 -2$

$\left(\sqrt[2]{x + 10}\right)^2 = (x-2)^2 \qquad (x-2)(x-2)$

$x + 10 = x^2 - 2x - 2x + 4$

$x + 10 = x^2 - 4x + 4$

$-x - 10 -x - 10$

$0 = x^2 - 5x - 6$

$0 = (x + 1)(x - 6)$

$\boxed{x = -1}$ (crossed out) $\boxed{x = 6}(x+1)(x-6)$

$2 + \sqrt{x + 10} = x$

Check

$2 + \sqrt{-1 + 10} = -1 \qquad\qquad 2\sqrt{6 + 10} = 6$

$2 + \sqrt{9} = -1 \qquad\qquad\quad 2\sqrt{16} = 6$

$2 + 3 = -1 \qquad\qquad\qquad 2 + 4 = 6\ \checkmark$

Mar 12-4:54 PM

Ⓑ $(x + 6)^{\frac{1}{2}} - (2x - 4)^{\frac{1}{2}} = 0$

$\sqrt{x + 6} - \sqrt{2x - 4} = 0$ Check

$\sqrt{x + 6}^2 = \sqrt{2x - 4}^2 \qquad \sqrt{10 + 6} = \sqrt{2(10) - 4}$

$x + 6 = 2x - 4 \qquad\qquad \sqrt{16} = \sqrt{20 - 4}$

$-x + 4 -x + 4 \qquad\qquad \sqrt{16} = \sqrt{16}\ \checkmark$

$\boxed{10 = x}$

6. Solve $(x + 5)^{\frac{1}{2}} - 2 = 1$.

$\sqrt{x + 5} - 2 = 1$

$ +2 = +2$

$\sqrt{x + 5}^2 = 3^2$

$x + 5 = 9 \qquad$ Check

$\boxed{x = 4} \qquad \sqrt{4 + 5} - 2 = 1$

$ \sqrt{9} - 2 = 1$

$ 3 - 2 = 1\ \checkmark$

Mar 12-5:00 PM

3

Solve the following, check for extraneous solutions

$$\left(2\sqrt{x}\right)^2 = \left(3\sqrt{x-2}\right)^2$$

$4x = 9(x-2)$ $x = \dfrac{18}{5}$

$4x = 9x - 18$

$\dfrac{-9x \quad -9x}{-5x = -18}$

Check

$2\sqrt{\dfrac{18}{5}} = 3\sqrt{\dfrac{18}{5} - 2}$

$2\dfrac{3\sqrt{2}}{\sqrt{5}} = 3\sqrt{\dfrac{18}{5} - \dfrac{10}{5}}$

$6\dfrac{\sqrt{2}}{\sqrt{5}} = 3\sqrt{\dfrac{8}{5}}$

$= 3\dfrac{2\sqrt{2}}{\sqrt{5}}$

$= 6\dfrac{\sqrt{2}}{\sqrt{5}}$

$$\sqrt{5x-11}^2 = (x-1)^2$$

$5x - 11 = x^2 - 2x + 1$

$\dfrac{-5x + 11 \qquad -5x + 11}{0 = x^2 - 7x + 12}$ factor

$0 = (x-3)(x-4)$ Check x=3

$x = 3, 4$

$\sqrt{15-11} = 3-1$

$\sqrt{4} = 2$ ✓

Check 4

$\sqrt{20-11} = 4-1$

$\sqrt{9} = 3$ ✓

$$\sqrt{2x+5} + 4 = 3$$

Mar 16-7:35 AM

Example 2   Solve the equation.

(A) $\sqrt[3]{x+2} + 7 = 5$

$\sqrt[3]{x+2}^3 = (-2)^3$

$x + 2 = -8$

$x = -10$

Check

$\sqrt[3]{-10+2} + 7 = 5$

$\sqrt[3]{-8} + 7 = 5$

$-2 + 7 = 5$ ✓

Mar 12-5:03 PM

4

B　　$\sqrt[3]{x-5} = x+1$

Your Turn

8.　Solve $2(x-50)^{\frac{1}{3}} = -10$.
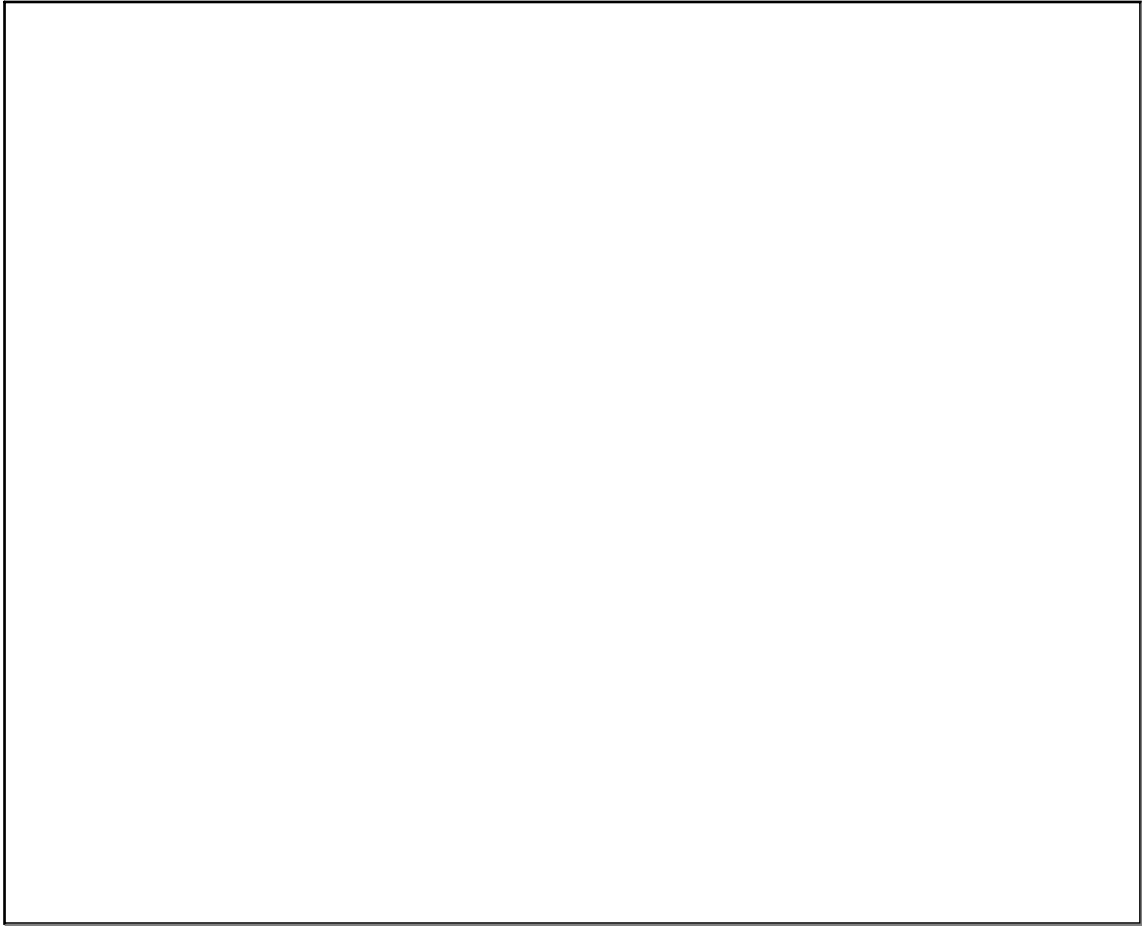
$$2\sqrt[3]{x-50} = -2$$

Mar 12-5:05 PM

Solve the following:

$$\sqrt[3]{x-5} = \sqrt[3]{7-x}$$

$$\sqrt[3]{x+2} = \sqrt[3]{x+3}$$

Mar 16-7:52 AM

Mar 19-10:16 PM