9-1 Defining and evaluating logarithms

I understand that the logarithm is the inverse of an exponential

I can verify an inverse function using composition

I can convert between logarithm and exponential form

Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.

Exponential Equation		Logarithmic Equation
$b^x = a$	$\Leftrightarrow$	$\log_b a = x$

base

$$b > 0, b \neq 1$$

base

Examples

Exponential Equation	Logarithmic Equation
$4^3 = 64$	$\log_4 64 = 3$
$5^{-2} = \frac{1}{25}$	$\log_5 \frac{1}{25} = -2$
$3^5 = 243$	$\log_3 243 = 5$
$4^{-3} = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$
$\left(\frac{3}{4}\right)^t = s$	$\log_{3/4} s = t$
$\frac{1}{5}^{w} = v$	$\log_{\frac{1}{5}} v = w$

$\log_{\frac{1}{5}} v = w$

The natural logarithm:

$y = \ln x$ is equivalent to $x = e^y$

e = natural #

$\log_e x = \ln x$

The common logarithm:

$y = \log x$ is equivalent to $x = 10^y$

$\log_{10} x = \log x$

Exponential Equation	Logarithmic Equation
$e^5 \approx 148.4$	$\ln(148.4) \approx 5$
$e^{1.8} \approx 6$	$\ln 6 \approx 1.8$
$10^5 = 100,000$	$\log 100,000 = 5$
$10^3 = 1,000$	$\log 1,000 = 3$

(A) If $f(x) = \log_{10} x$, find $f(1000)$, $f(0.01)$, and $f(\sqrt{10})$.

$f(1000) =$

$\log_{10} 1000 = ? = \boxed{3}$

$10^? = 1000$

$f(0.01) =$

$\log_{10}(.01) = \boxed{-2}$

$0.01 = \frac{1}{100}$

$10^? = \frac{1}{100}$

$f(\sqrt{10}) = \boxed{\frac{1}{2}}$

$\log_{10} \sqrt{10}$

$10^? = \sqrt{10}$

$10^? = 10^{1/2}$

(B) If $f(x) = \log_{\frac{1}{2}} x$, find $f(4)$, $f\left(\frac{1}{32}\right)$ and $f(2\sqrt{2})$.

$f(4) =$

$f\left(\frac{1}{32}\right) =$

$f(2\sqrt{2}) =$

Find the exact value without a calculator

$\log_2 32 = \boxed{5}$

$2^? = 32$

$2^5 = 32$

$\log_4 \frac{1}{16} = \boxed{-2}$

$4^{-2} = \frac{1}{16}$

$\log 10{,}000{,}000 = 7$

$10^? = 10{,}000{,}000$

$\log .00001 = \boxed{-5}$

$10^? = .00001$

You try

$\log_5 25 = 2$

$\log_2 \frac{1}{8} = -3$

$\log 1000 = 3$

$\log .001 = -3$

The acidity level, or pH, of a liquid is given by the formula $pH = \log \frac{1}{[H^+]}$ where $[H^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from 1.58×10^{-8} moles per liter to 6.31×10^{-8} moles per liter. What is the range of the pH for a typical swimming pool?

pH = acidity

$[H^+]$ = hydrogen-ion concentration

$$pH = \log\left(\frac{1}{1.58 \times 10^{-8}}\right) = 7.8$$

$$pH = \log\left(\frac{1}{6.31 \times 10^{-8}}\right) = 7.2$$

The intensity level L (in decibels, dB) of a sound is given by the formula $L = 10 \log \frac{I}{I_0}$ where I is the intensity (in watts per square meter, W/m^2) of the sound and I_0 is the intensity of the softest audible sound, about 10^{-12} W/m^2. What is the intensity level of a rock concert if the sound has an intensity of 3.2 W/m^2?