# 9-1 Defining and evaluating logarithms

I understand that the logarithm is the inverse of an exponential

I can verify an inverse function using composition

I can convert between logarithm and exponential form

## Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.

| Exponential Equation | Logarithmic Equation |
| --- | --- |
| $b^x = a$ | $\log_b a = x$ |
| base | base |
| $\log_b a = x$ | $b^x = a$ |

$$b > 0, b \neq 1$$

Examples

| Exponential Equation | Logarithmic Equation |
|---|---|
| $4^3 = 64$ | $\log_4 64 = 3$ |
| $5^{-2} = \frac{1}{25}$ | $\log_5 \frac{1}{25} = -2$ |
| $3^5 = 243$ | $\log_3 243 = 5$ |
| $4^{-3} = \frac{1}{64}$ | $\log_4 \frac{1}{64} = -3$ |
| $\left(\frac{3}{4}\right)^t = s$ | $\log_{3/4} s = t$ |
| $\left(\frac{1}{5}\right)^w = v$ | $\log_{\frac{1}{5}} v = w$ |

The natural logarithm:

$y = \ln x$ is equivalent to $x = e^y$

$\ln_e x = y$ $e^y = x$

e = natural number $\ln x = y$

The common logarithm:

$y = \log_{10} x$ is equivalent to $x = 10^y$

| Exponential Equation | Logarithmic Equation |
|---|---|
| $e^5 \approx 148.4$ | $\ln 148.4 \approx 5$ |
| $e^{1.8} \approx 6$ | $\ln 6 \approx 1.8$ |
| $10^5 = 100{,}000$ | $\log 100{,}000 = 5$ |
| $10^3 = 1000$ | $\log 1{,}000 = 3$ |

Ⓐ If $f(x) = \log_{10} x$, find $f(1000)$, $f(0.01)$, and $f(\sqrt{10})$.

$f(1000) =$

$\log_{10} 1000 = ? = 3$

$10^? = 1000$

$10^3 = 1000$

$f(0.01) =$

$\log_{10}(0.01) = ? = -2$

$10^? = .01$

$10^{-2} = .01$

$f(\sqrt{10}) = \frac{1}{2}$

$\log_{10} \sqrt{10}$

$\sqrt{10} = 10^{1/2}$

$10^? = 10^{1/2}$

Ⓑ If $f(x) = \log_{\frac{1}{2}} x$, find $f(4)$, $f\left(\frac{1}{32}\right)$ and $f(2\sqrt{2})$.

$f(4) =$

$\log_{\frac{1}{2}}(4) = -2$

$\frac{1}{2}^? = 4$

$f\left(\frac{1}{32}\right) =$

$\log_{\frac{1}{2}}\left(\frac{1}{32}\right) = 5$

$\left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \frac{1}{32}$

$f(2\sqrt{2}) = 3/2$

$2\sqrt{2}$

$2^1 \cdot 2^{1/2}$

$2^{3/2}$

Find the exact value without a calculator

$\log_2 32 = 5$

$2^? = 32$

$2^5 = 32$

$\log_4 \frac{1}{16} = \boxed{-2}$

$4^{-2} = \frac{1}{16}$

$\log 10{,}000{,}000 = 7$

$10^? = 10{,}000{,}000$

$10^{⑦} = 10{,}000{,}000$

$\log .00001 = -5$

$10^? = .00001$

$10^{-5}$

You try

$\log_5 25 = 2$

$\log_2 \frac{1}{8} = -3$

$\log 1000 = 3$

$\log .001 = -3$

$f(243) = \log_3 243$

$f(\frac{1}{27}) = \log_3 (\frac{1}{27})$

$f(\sqrt{27}) = \log_3(\sqrt{27}) = 3/2$

$3^? = \sqrt{27} = \sqrt{3^3} = (3^3)^{1/2} = 3^{3/2}$

$3^? = 3^{3/2}$

The acidity level, or pH, of a liquid is given by the formula $pH = \log \frac{1}{[H^+]}$ where $[H^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from $1.58 \times 10^{-8}$ moles per liter to $6.31 \times 10^{-8}$ moles per liter. What is the range of the pH for a typical swimming pool?

acidity

pH = acidity

$[H^+]$ = Hydrogen Ions

$[H^+] = 1.58 \times 10^{-8}$

$[H^+] = 6.31 \times 10^{-8}$

$pH = \log\left(\frac{1}{1.58 \times 10^{-8}}\right) = 7.8$

$pH = \log\left(\frac{1}{(6.31 \times 10^{-8})}\right) = 7.2$

The intensity level $L$ (in decibels, dB) of a sound is given by the formula $L = 10 \log \frac{I}{I_0}$ where $I$ is the intensity (in watts per square meter, $W/m^2$) of the sound and $I_0$ is the intensity of the softest audible sound, about $10^{-12}$ $W/m^2$. What is the intensity level of a rock concert if the sound has an intensity of 3.2 $W/m^2$?

$$L = 10 \log \frac{I}{10^{-12}}$$

? → $L$

$I$ ← intensity ← 3.2

$10^{-12}$ ← intensity of softest

$$10 \cdot \log\left(\frac{3.2}{10^{-12}}\right) =$$

$$L(x) = 10 \cdot \log \frac{x}{10^{-12}}$$

$L$ ↑ loudness

$x$ ← intensity

whisper intensity = $10^{-10}$