9-2: Properties of Logarithms

I can understand the properties of logarithms and use them to simplify logs.

I can apply multiple properties to a single logarithm.

$\log_6 36 = \cancel{?} = 2$

$6^2 = 36$

Find the value of each logarithm without using a calculator.

1. $\log_7 7 = x = 1$

   $7^x = 7$

2. $\log_{18} 18 = 1$

3. $\log_5 1 = x = 0$

   $5^x = 1$

4. $\log_9 1 = 0$

$$\log_a 1 = \underline{\underline{0}} \qquad \log_a a = 1$$

any base

match

Evaluate

$\log_5 1 = 0$

$\ln \underline{1} = 0$

$\log_4 4 = 1$

$\log_{10} 10 = 1$

Evaluate the logarithm:

1. $\log_3(3^2) = x = 2$ $\qquad 3^x = 3^2$

2. $\log_5(5^8) = 8$

Without evaluating, predict what the following logs equal:

3. $\cancel{\log_2} 2^{10} = 10$

4. $\log_{20} 20^7 = 7$

Inverse Property of Logarithms
If b and r are positive real numbers, with $b \neq 0$, then

$$\cancel{\log_a a}^r = r$$

Evaluate

$\cancel{\log_4 4}^3 = 3$

$\cancel{\ln e}^{-0.5} = -0.5$

Recall: $b^x = a \iff \log_b a = x$

How would we write the following exponential as a log?

exponential $\cancel{5^{\log_5}}(20) = x = 20$

logarithm: $\log_5 x = \log_5 20$

Inverse Property of Logarithms

If b and M are positive real numbers, with $b \neq 0$, then

$\log_{20} 20^7$

$$\cancel{b^{\log_b}} M = M$$

Evaluate

$\cancel{5^{\log_5}} 20 = 20$

$\cancel{8^{\log_8}} \sqrt{23} = \sqrt{23}$

$\cancel{12^{\log_{12}}} \sqrt{2} = \sqrt{2}$

$\cancel{10^{\log_{10}}} 0.2 = 0.2$

Exponent Rules Review

$2^5 \cdot 2^3 = 2\cdot2\cdot2\cdot2\cdot2 \cdot 2\cdot2\cdot2 = 2^{5+3} = 2^8$

$\dfrac{2^5}{2^3} = \dfrac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = 2^{5-3} = 2^2$

$(2^3)^5 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2)\cdot(2\cdot2\cdot2)\cdot(2\cdot2\cdot2)\cdot(2\cdot2\cdot2)$

$= 2^{3\cdot5} = 2^{15}$

$\sqrt[3]{8} = 8^{1/3}$ (fraction exponent)

powers in sky, roots in the ground

$2^{-1} = \dfrac{1}{2}$ (negative exponents flip it)

Product Rule of Logarithms

If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(\underbrace{MN}_{\text{multiply}}) = \log_b M \underbrace{+}_{\text{add}} \log_b N$$

Which exponent rule is this similar to?

$2^3 \cdot 2^5 = 2^{3+5} = 2^8$

Why would we want to be able to split up a logarithm?

Write each of the following logarithms as the sum of logarithms.

$\log_2(5 \cdot 3)$

$\log_2(5) + \log_2(3)$

$\ln(6z)$

$\ln(6) + \ln(z)$

$\log_4(9 \cdot 5)$

$\log_4(9) + \log_4(5)$

$\log(5w)$

$\log(5) + \log(w)$

How do you predict we would write the following logarithm as two logarithms?

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

Quotient Rule of Logarithms

If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_2\left(\frac{5}{3}\right) = \log_2(5) - \log_2(3)$$

$$\log\left(\frac{y}{5}\right) = \log(y) - \log(5)$$

$$\log_7\left(\frac{9}{5}\right) = \log_7(9) - \log_7(5)$$

$$\ln\left(\frac{p}{3}\right)$$

Write the following as the sum or difference of logarithms.

(expand the logarithm)

$$\log_3\left(\frac{4x}{y}\right) = \log_3(4) + \log_3(x) - \log_3(y)$$

$$\log_3\left(\frac{3m}{n}\right) = \log_3(3) + \log_3(m) - \log_3(n)$$

$$\log_3\left(\frac{q}{3p}\right) = \log_3(q) - \log_3(3) - \log_3(p)$$

Show that the following logs are equal:

$$\log_2(4)^3 = 3 \cdot \log_2 4$$

$$\log_2(4^3)$$

$$\log_2(4 \cdot 4 \cdot 4)$$

$$\log_2 4 + \log_2 4 + \log_2 4 = 3 \cdot \log_2 4$$

Power Rule of Logarithms
If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r \log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$\log_8 3^5$

$5 \cdot \log_8 3$

$\ln x^{\sqrt{3}}$ $\sqrt{3} \cdot \ln x$

$\log_2 5^{1.6} = 1.6 \cdot \log_2 5$

$\log b^5 = 5 \log b$

Expand the logarithm.

$\log_2(x^2 y^3)$

$\log_2 x^2 + \log_2 y^3$

$2 \cdot \log_2 x + 3 \cdot \log_2 y$

$\log\left(\frac{100x}{\sqrt{y}}\right) =$

$\log 100 + \log x - \log y^{1/2}$

$\log 100 + \log x - \frac{1}{2} \log y$

$\log_4(a^2 b)$

$\log_3\left(\frac{9m^4}{\sqrt[3]{n}}\right)$

$\log_3 9 + \log_3 m^4 - \log_3 n^{1/3}$

$\log_3 + 4 \log_3 m - \frac{1}{3} \log_3 n$

Write each of the following as a single logarithm.

$\log_6 3 + \log_6 12 = \log_6(3 \cdot 12) = \log_6(36)$

$\log(x-2) - \log x = \log\left(\frac{x-2}{x}\right)$

$\log_5 x - 3\log_5 2^3 = \log_5\left(\frac{x}{2^3}\right) = \log_3\left(\frac{x}{8}\right)$

$\log(x-1) + \log(x+1) - 3\log x$

Rewrite and express in terms of a and b given that a=ln3 and b=ln4

ln12

ln16

How do we evaluate logs in a calculator??

Change of Base Formula

If $a \neq 0$, $b \neq 0$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

which means:

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

Use your calculator to approximate the following:

$$\log_4 45 = \frac{\log(45)}{\log(4)} = 2.75$$

$$\log_3 75 = \frac{\ln(75)}{\ln(3)} = \frac{\log(75)}{\log(3)}$$

same

$$\log_6 40$$

$$= \frac{\log(40)}{\log(6)} =$$