9-3 Solving Exponential and Logarithmic equations

Objectives:

I can solve exponential and logarithmic equations both graphically and algebraically.

Solving Graphically

$$275e^{0.06x} = 1000$$

$y_1 = 275e^{.06x}$ $y_2 = 1000$

$x = 21.52$

$$10^{2x} = 1500$$

$y_1 =$ $y_2 =$

$x = 1.59$ $x = 1.58880\ldots$

Inverses

Addition/Subtraction	Natural Log/e^	Common Log/10^	Log base b/b^
$x-5=10$ $+5 \quad +5$ $x=15$	$\ln e^x = \ln 5$ base $\ln e^x = \ln 5$ $x = \ln(5)$	$\log 10^x = \log 100$ $\log_{10} 10^x = \log 100$ $x = \log 100$	$\log_2 2^x = \log_2 16$ $\log_2 2^x = \log_2 16$ $x = \log_2 16$
$x+7=21$ $-7 \quad -7$ $x=14$	$\ln x = 7$ $e \quad e$ $x = e^7$	$\log x = 3$ $10 \quad 10$ $x = 10^3$	$\log_3 x = 4$ $3 \quad 3$ $x = 3^4$

Solve the following equations

$\frac{10}{5} = \frac{5e^{4x}}{5}$

$\ln 2 = \ln e^{4x}$

$\frac{\ln(2)}{4} = \frac{4x}{4}$

$x = 0.17$

$\log_3(2x-4) = 4$

$2x-4 = 3^4$

$2x-4 = 81$
 $+4 \quad +4$

$\frac{2x}{2} = \frac{85}{2}$

$x = 42.5$

$5^{x-1} - 4 = 7$
 $+4 \quad +4$

$\log_5 5^{x-1} = \log_5 11$

$x-1 = \log_5(11)+1$
 $+1$

$x = \log_5(11)+1$

$\log(11)/\log(5)+1$

$x = 2.49$

$\log_6 6^{3x} = \log_6 12$

$3x = \log_6(12)$

$\frac{3x}{3} = \frac{\log_6(12)}{3}$

$x = \frac{\log_6(12)}{3}$

$\log(12)/\log(6)/3$

$x = 0.46$

Solve the following

$\ln(x+12) = 3\ln 2$

$\log(x^4) = 2$

$\sqrt[4]{x^4} = \sqrt[4]{100}$

$x = \sqrt[4]{100} = 3.16$

$\log x^4 = 2$

$\frac{4 \cdot \log x}{4} = \frac{2}{4}$

$\log x = \frac{1}{2}$

$x = 10^{1/2}$

$4\ln(x+7) - 5 = 1$

$+5 \quad +5$

$\frac{4\ln(x+7)}{4} = \frac{6}{4}$

$e^{\ln(x+7)} = e^{1.5}$

$x + 7 = e^{1.5}$

$-7 \quad -7$

$x = e^{1.5} - 7 = e \wedge (1.5) - 7$

$= -2.52$

$\ln(x+12) = 3\ln 2$

$\log_b M^r = r \log_b M$

$\ln(x+12) = \ln(2^3)$

$\ln(x+12) = \ln(8)$

$e \quad e$

$x + 12 = 8$

$-12 \quad -12$

$x = -4$

Solve the following

$\log(x+2)-3=5$

$+3 \quad +3$

$\log_{10}(x+2)=8$

$10^{\log_{10}(x+2)}=10^{8}$

$x+2=100{,}000{,}000$

$-2 \quad -2$

$x=99{,}999{,}998$

$\log_4(1-x)=1$

$4^{\log_4(1-x)}=4^{1}$

$1-x=4$

$-1 \quad -1$

$-1x=3$

$\frac{-1x}{-1}=\frac{3}{-1}$

$x=-3$

Suppose that $250 is deposited into an account that pays 4.5% compounded quarterly. Solve for t to find how long it will take for the account to contain at least $500.

$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$

$n=4$

$500=250\left(1+\frac{.045}{4}\right)^{4t}$

$\frac{500}{250}=\frac{250(1.01125)^{4t}}{250}$

$\log_{1.01125}(2)=\log_{1.01125}\left((1.01125)^{4t}\right)$

$\frac{\log_{1.01125}(2)}{4}=\frac{4t}{4}=15.5 \text{ yrs}$

How long will it take for a $250 initial investment in an account that pays 4.5% compounded continuously to grow to $750?

P

r

$A(t) = Pe^{rt}$

A(t)

$750 = 250e^{.045t}$

$\frac{750}{250} = \frac{250e^{.045t}}{250}$

$\ln(3) = \ln e^{.045t}$

$\frac{\ln(3)}{.045} = \frac{.045t}{.045} = 24.4 \text{ yrs}$

Comparing acidity: $pH = -\log\left[H^+\right]$

$\left[H^+\right]$ = hydrogen-ion concentration

Vinegar has a pH of 2.4. What is it's hydrogen ion concentration?

Baking soda has a pH of 8.4. What is it's hydrogen ion concentration?

Which has a higher hydrogen ion concentration?