# Serving Unit-Circle Trigonometry on a Paper Plate 

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Time needed: Most of 1 class period to create triangles and find degree measures. A part of another day to find and label the radian measurements.

Materials needed for each student: cheap paper plate, 3 sheets of colored paper with a circle of 14.5 cm diameter copied on each paper (the size of the center circle on the paper plate), ruler, glue stick, scissors, and either 4 colors of markers OR stickers (if using stickers you'll need 4 different colors, 4 of each color +1 extra of one of the colors =17). It works really well 3 colors of the stickers/markers match the colored paper.

1) Create 45 degree piece:
a) Cut out a circle of the first color. Fold in half, in half again, and in half again. Unfold. Ask: What is the measure of the central angles created? (45 degrees) Why do we know it is 45 degrees?
b) Place the circle on the center of the paper plate, and use the folds as a guide to make the x \& y axis through the center of the paper plate, set the plate aside.
c) Refold so you have a quarter circle. You will see one bisecting fold. Hold so that the solid folded edge is up and the open flaps down, cut along the fold almost to the point, then angle your scissors upwards and cut off the upper half. This will leave a little connecting piece. (When you unfold, and it will look like a bow tie with a tiny center.) Use a ruler to drop a perpendicular from one outside corner down to the radius, and cut off the curved part of the circle, so it looks like a true triangle.

Ask: What kind of triangle is this? (isosceles right triangle) Why do we
 know it is isosceles? Why do we know it is a right triangle? Which sides are equal?

Ask: If we call the hypotenuse length = 1, what is the length of the legs of the triangle? Rationalize the denominator.
d) Glue the "bow tie" onto the center of the paper plate, so the $x$ axis is going through the length of the bowtie. Redraw the $x$-axis so you can see it. On the paper plate, label the ( $\mathrm{x}, \mathrm{y}$ ) points in each quadrant at the edges of the bow tie, using the distances found. (NOTE: Write close to the bow tie, later you will be writing (or using stickers) to label the radian measurements, so need space on rippling edges of the paper plate.)
2) Create 60 degree piece:
a) Cut out a circle of the second color. Fold in half. CAREFULLY make two more folds, dividing the half circle into $1 / 3$ sections (ice cream cone). It works best if you fold one section forward and
one section back. Fold the ice cream cone in half. Unfold. Ask: What is the measure of the central angles created? ( 60 degrees) Why do we know they are 60 degrees?
b) Refold into a quarter circle. You will see two trisecting folds. Hold the quarter circle so the solid fold is up and the flaps are down. Cut along the upper fold almost to the center, then angle the scissors up and cut off the upper 1/3 of the paper, leaving a little connecting piece. (If you unfolded it, it would be a bowtie.)


Using a ruler, drop a perpendicular from perimeter of the upper cut-off fold, down to the radius. Cut off the perpendicular, removing curved part of the circle.
c) Ask: What type of triangle is this? (30-60-90) How do we know? Which edge is the radius of the original circle? (Hypotenuse) If the length of the hypotenuse is 1 , what are the lengths of each of the legs?
d) Unfold, and glue the bow tie on to the paper plate. Label the ( $\mathrm{x}, \mathrm{y}$ ) points on the bow tie, using the lengths of the legs of the triangle. Redraw the x-axis.
3) Create the 30 degree angle piece: Follow directions as above. Instead of cutting on the top fold, cut on the lower fold. Be sure to leave a small connecting piece for the bow tie. Glue to paper plate and label the ( $\mathrm{x}, \mathrm{y}$ ) points of the bow tie.


Finish by adding degree markings at each axis and in each quadrant, on the very outer edges of the plate. Leave room to add radians on the ribbed part of the plate.

## Radians

When ready to discuss radian measurements, use stickers or colored markers to label the radian measurements on the outer edge of the paper plate. Use two stickers at the 0,360 position.

