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Describe how the graph of  $g(x)$  is related to the graph  $f(x) = \frac{1}{x}$  or  $f(x) = \frac{1}{x^2}$ .

1.  $g(x) = \frac{5}{x} - 3$   $\frac{1}{x}$   
 • V. Stretch by 5  
 • Shift Down 3

2.  $g(x) = \frac{-1}{x} + 5$   $\frac{1}{x}$   
 • V. Flip  
 • Shift up 5

3.  $g(x) = -\frac{1}{(x-2)^2} + 4$   $\frac{1}{x^2}$   
 • V. Flip  
 • Shift Right 2  
 • Shift Up 4

Use synthetic division to put into transformation form, and then state the transformations.

4.  $f(x) = \frac{3x+5}{x-1} = \frac{8}{x-1} + 3$   
 $\begin{array}{r} 3 \ 5 \\ \downarrow 3 \\ 3 \ 8 \end{array}$   
 • V. Stretch by 8  
 • Shift Right 1  
 • Shift Up 3

5.  $f(x) = \frac{x-2}{x+4} = \frac{-6}{x+4} + 1$   
 $\begin{array}{r} 1 \ -2 \\ \downarrow -4 \\ 1 \ -6 \end{array}$   
 • V. Flip  
 • V. Stretch by 6  
 • Shift Left 4  
 • Shift Up 1

6.  $f(x) = \frac{2x-7}{x+1} = \frac{-9}{x+1} + 2$   
 $\begin{array}{r} 2 \ -7 \\ \downarrow -2 \\ 2 \ -9 \end{array}$   
 • V. Flip  
 • V. Stretch by 9  
 • Shift Left 1  
 • Shift Up 2

Given the following functions, find all holes, vertical asymptotes, x and y intercept and HEB.  
H.A.

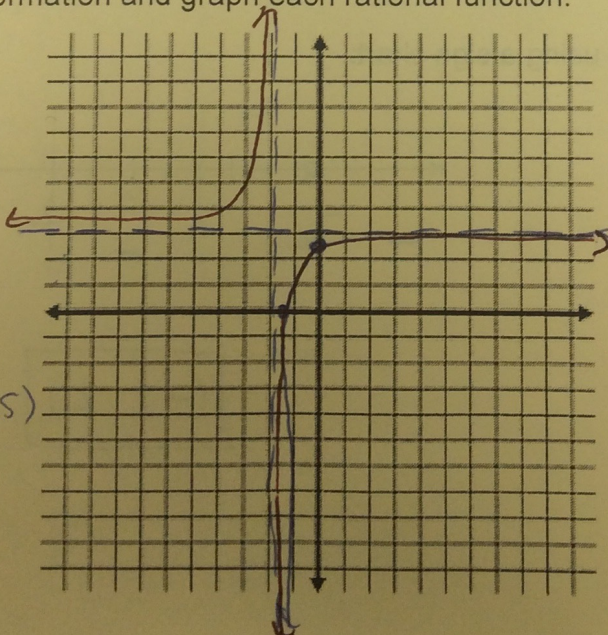
8.  $f(x) = \frac{x-3}{x^2+6x+5} = \frac{(x-3)}{(x+1)(x+5)}$   $\frac{\text{deg } 1}{\text{deg } 2}$   
 Holes: None  
 VA:  $x = -1, -5$   
 X-int:  $(3, 0)$   
 y-int:  $(0, -3/5)$   
 H.A.  $y = 0$  (Bottom Heavy)

9.  $f(x) = \frac{(x-2)(x+2)}{(x+1)}$   
 Holes: None  
 VA:  $x = -1$   
 X-int:  $(2, 0), (-2, 0)$   
 y-int:  $(0, -4)$   
 H.A.  $y = \text{slant asymptote}$  (Top Heavy)

Find the following information and graph each rational function:

10.  $f(x) = \frac{3x+5}{x+2}$

Holes: None  
 VA:  $x = -2$   
 x-int:  $(-5/3, 0)$   
 y-int:  $(0, 5/2)$  or  $(0, 2.5)$   
 HA:  $y = \frac{3}{1} = 3$



Domain:  $(-\infty, -2) \cup (-2, \infty)$   
 Range:  $(-\infty, 3) \cup (3, \infty)$   
 Increasing:  $(-\infty, -2) \cup (-2, \infty)$   
 Decreasing: None  
 End Behavior:  
 $\lim_{x \rightarrow -\infty} f(x) = 3$      $\lim_{x \rightarrow +\infty} f(x) = 3$

$$11. f(x) = \frac{x-3}{(x-4)(x+1)}$$

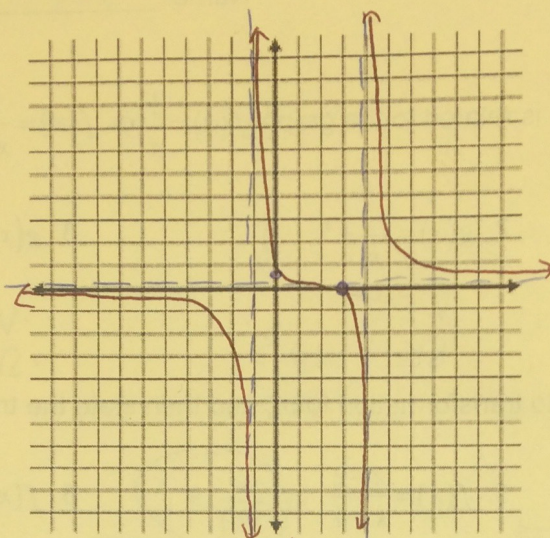
Holes: None

VA:  $x=4, x=-1$

x-int:  $(3,0)$

y-int:  $(0, 3/4)$

HA:  $y=0$



Domain:  $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$

Range:  $(-\infty, \infty)$

Increasing: None

Decreasing:  $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$

End Behavior:

$$\lim_{x \rightarrow -\infty} f(x) = 0 \quad \lim_{x \rightarrow +\infty} f(x) = 0$$

$$12. f(x) = \frac{x^2 - 2x - 3}{x^2 - 1}$$

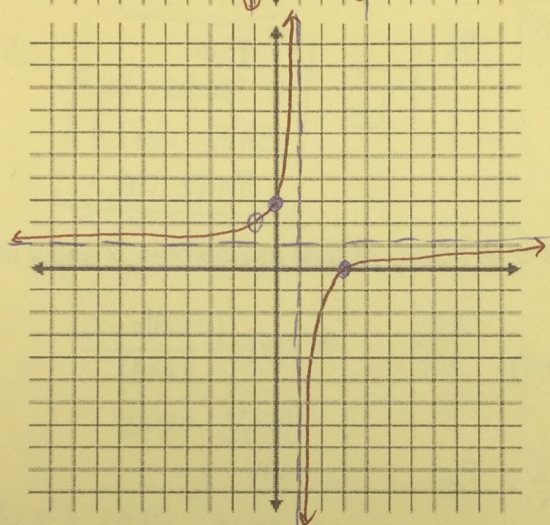
Holes:  $\frac{(x-3)(x+1)}{(x-1)(x+1)}$   
 $x=-1$

VA:  $x=1$

x-int:  $(3,0)$

y-int:  $(0, 3)$

HA:  $y=1$



Domain:  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range:  $(-\infty, 1) \cup (1, \infty)$

Increasing:  $(-\infty, 1) \cup (1, \infty)$

Decreasing: None

End Behavior:

$$\lim_{x \rightarrow -\infty} f(x) = 1 \quad \lim_{x \rightarrow +\infty} f(x) = 1$$

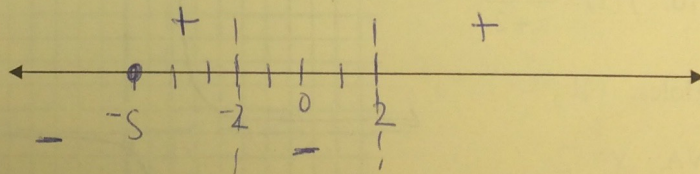
Solve the following inequalities using a sign chart:

$$13. \frac{x+5}{x^2-4} < 0 \quad \text{Below w/ Round}$$

$$\frac{x+5}{(x+2)(x-2)} < 0$$

VA:  $x=-2, 2$

x-int:  $(-5, 0)$



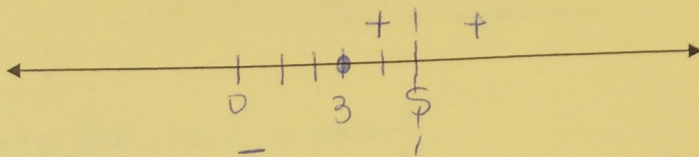
$$(-2, 2)$$

$$\frac{x-3}{(x-5)(x-5)} \geq 0$$

14.  $\frac{x-3}{x^2-10x+25} \geq 0$  Above w/ square on x-int

x-int: (3, 0)

VA: x=5



~~$(-\infty, 3] \cup (5, \infty)$~~   $[3, 5) \cup (5, \infty)$

15. If the degree is the same in the numerator and denominator, the asymptote is found by dividing the leading coefficients

16. If the degree in the denominator is larger than the numerator, they asymptote will be at

~~if horizontal asymptote~~  
 $y=0$

bottom heavy ~~top heavy~~

17. If the degree in the numerator is larger than the degree in the denominator, the asymptote will be a y = slant asymptote

denominator  
 top heavy