

Secondary III
Review Unit 1

Name Key
Class A3 & B7

Unit 1:
Fill in each blank with the graph that corresponds to each of the parent functions given below.

1. $f(x) = x$ C

2. $f(x) = x^2$ B

3. $f(x) = |x|$ D

4. $f(x) = \sqrt{x}$ A

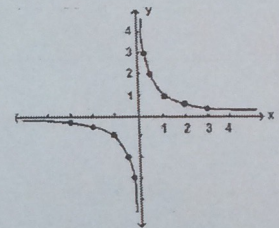
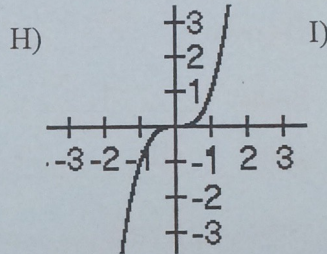
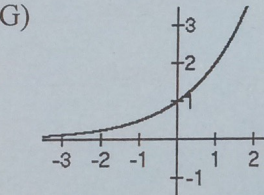
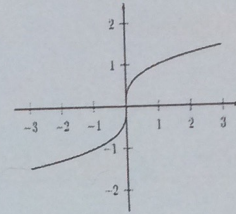
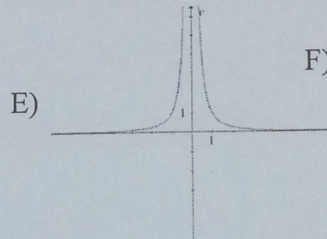
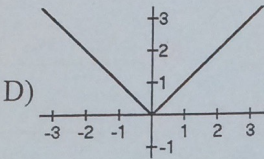
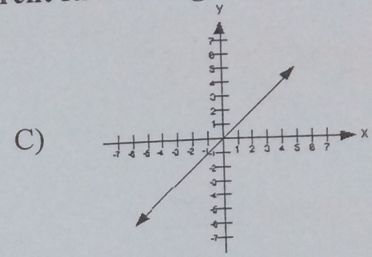
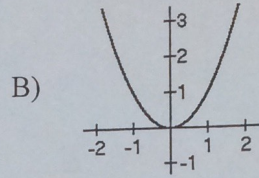
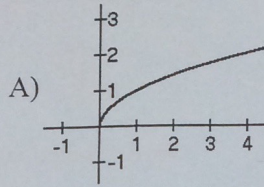
5. $f(x) = x^3$ H

6. $f(x) = \frac{1}{x}$ I

7. $f(x) = 2^x$ G

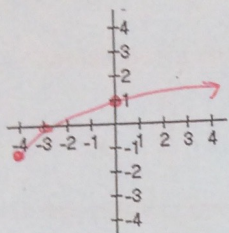
8. $f(x) = \sqrt[3]{x}$ F

9. $f(x) = \frac{1}{x^2}$ E



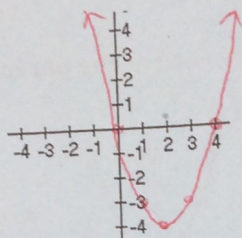
Graph the following functions, identify the parent function, and list the transformations involved.

10. $g(x) = \sqrt{x+4} - 1$



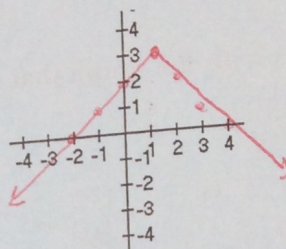
Parent Function: \sqrt{x}

11. $h(x) = (x-2)^2 - 4$



Parent Function: x^2

12. $f(x) = -|x-1| + 3$



Parent Function: $|x|$

List the transformations in words:

- a) Shift Left 4
- b) Shift Down 1

List the transformations in words:

- a) Shift Right 2
- b) Shift Down 4

List the transformations in words:

- a) V. Flip
- b) Shift Right 1
- c) Shift Up 3

Domain: $[-4, \infty)$

Range: $[-1, \infty)$

x-int: $(-3, 0)$

y-int: $(0, 1)$

Left EB: None
 ~~As $x \rightarrow -\infty$, $y \rightarrow -\infty$~~

Right EB: As $x \rightarrow +\infty$, $y \rightarrow +\infty$

Inc: $(-4, \infty)$

Dec: None

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

x-int: $(0, 0), (4, 0)$

y-int: $(0, 0)$

Left EB: As $x \rightarrow -\infty$, $y \rightarrow +\infty$

Right EB: As $x \rightarrow +\infty$, $y \rightarrow +\infty$

Inc: $(2, \infty)$

Dec: $(-\infty, -2)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

x-int: $(-2, 0), (4, 0)$

y-int: $(0, 2)$

Left EB: As $x \rightarrow -\infty$, $y \rightarrow -\infty$

Right EB: As $x \rightarrow +\infty$, $y \rightarrow -\infty$

Inc: $(-\infty, 1)$

Dec: $(1, \infty)$

Unit 2:

1. Write the following polynomial in standard form: $8x^3 + 7x - 14x^{10} + 4 - 6x^9 + 2x^{11}$

$$2x^{11} - 14x^{10} - 6x^9 + 8x^3 + 7x + 4$$

Complete the polynomial operations and write the answers in standard form.

2. $(82x^8 + 21x^2 - 6) + (18x + 7x^8 - 42x^2 + 3)$

$$89x^8 - 21x^2 + 18x - 3$$

3. $(3x^3 - 4x^2 + 10) - (x^3 + 2x^2 + 5x)$

$$\begin{array}{r} 3x^3 - 4x^2 + 10 \\ -x^3 - 2x^2 - 5x \\ \hline 2x^3 - 2x^2 - 5x + 10 \end{array}$$

4. $(x+2)(3x-7)$

$$3x^2 - 7x + 6x - 14$$

$$3x^2 - x - 14$$

5. $(15x - 12x^{12} + x^9 + x^7 + 3x^2) + (x^7 - 68x^2 - x^9)$

$$-12x^{12} + 2x^7 - 65x^2 + 15x$$

6. $(6x^5 + 9x^3 - 17x + 5) - (5x^4 + 2x^3 + 7x + 5)$

$$6x^5 - 5x^4 + 11x^3 - 24x + 10$$

7. $(x^3 + 7x^2 - 14x + 3) \div (x - 2)$

$$\begin{array}{r} x^2 + 9x + 4 \quad R: 11 \\ x-2 \overline{) x^3 + 7x^2 - 14x + 3} \\ \underline{-(x^3 - 2x^2)} \\ 9x^2 - 14x \\ \underline{-(9x^2 - 18x)} \\ 4x + 3 \\ \underline{-(4x - 8)} \\ 11 \end{array}$$

$$\begin{array}{r} 1 \ 7 \ -14 \ 3 \\ \underline{-2 \ 18 \ 8} \\ 1 \ 9 \ 4 \ 11 \end{array}$$

$$x^2 + 9x + 4 \quad R: 11$$

8. $(2x-1)(x^2+3x-4)$

$$\begin{array}{r} 2x^3 + 6x^2 - 8x \\ -x^2 - 3x + 4 \\ \hline 2x^3 + 5x^2 - 11x + 4 \end{array}$$

$$2x^3 + 5x^2 - 11x + 4$$

9. $(2x^3 + 5x - 8) \div (x + 3)$

$$\begin{array}{r} 2x^2 - 6x - 13 \quad R: 31 \\ -3 \overline{) 2 \ 0 \ 5 \ -8} \\ \underline{-6 \ -18 \ 39} \\ 2 \ -6 \ -13 \ 31 \end{array}$$

$$2x^2 - 6x - 13 \quad R: 31$$

Factor the polynomial.

10. $3x - 9$

$$3(x-3)$$

11. $5x^2 - 25x$

$$5x(x-5)$$

12. $x^2 + 5x + 4$

$$(x+4)(x+1)$$

13. $3x^2 - 18x + 27$

$$3(x^2 - 6x + 9)$$

$$3(x-3)(x-3)$$

14. $x^2 - 10x + 21$

$$(x-7)(x-3)$$

15. $(7x^3 + x^2) - 28x - 4$

$$(7x+1)(x^2-4) \rightarrow (7x+1)(x+2)(x-2)$$

$$x^2(7x+1) - 4(7x+1)$$

Unit 3:

1. Use division to determine if $(x+2)$ or $(x-6)$ are factors of $f(x) = 2x^3 + 8x^2 - 22x - 60$.

$$\begin{array}{r} -2 \overline{) 2x^3 + 8x^2 - 22x - 60} \\ \underline{2x^3 + 4x^2} \\ 4x^2 - 22x - 60 \\ \underline{4x^2 + 8x} \\ -30x - 60 \\ \underline{-30x - 60} \\ 0 \end{array}$$

$(x-6)$ isn't a factor!

$$\begin{array}{r} 6 \overline{) 2x^3 + 8x^2 - 22x - 60} \\ \underline{12x^2} \\ 2x^2 + 98x - 60 \\ \underline{2x^2 + 120x} \\ -22x - 60 \\ \underline{-22x - 132} \\ 72 \end{array}$$

2. When you divide a polynomial by $(x-a)$, then $(x-a)$ is a factor if the remainder equals 0.

Find all the **zeros** and **factors** of the following functions:

3. $g(x) = x^3 + 4x^2 + 4x$

$x(x^2 + 4x + 4)$
 $x(x+2)(x+2)$ ← factors
 $x = 0, -2$ ← zeros
 ↑
 multiplicity 2

5. $g(x) = x^2 + 3x - 10$

$(x+5)(x-2)$ ← factors
 $x = -5, 2$ ← zeros

4. $h(x) = x^3 - 4x^2 - 5x$

$x(x^2 - 4x - 5)$
 $x(x-5)(x+1)$ ← factors
 $x = 0, 5, -1$ ← zeros

6. $f(x) = x^4 - 10x^2 + 9$

zeros $x = -3, -1, 1, 3$ (use a calculator or graph)
 factors → $(x+3)(x+1)(x-1)(x-3)$

Given the following zeros and multiplicities, write a function in factored form:

7. 3 (multiplicity of 2), -5, -7

$(x-3)^2(x+5)(x+7)$
 ↑
 exponent

8. 2, -4 (multiplicity of 5), 3 (multiplicity of 2)

$f(x) = (x-2)(x+4)^5(x-3)^2$

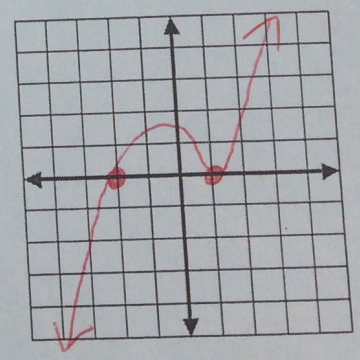
For the following functions, find the zeros, state the multiplicity of each zero and the type of intersection. State the end behavior and sketch a graph by hand.

9. $f(x) = (x+2)(x-1)^2$

Zeros	Multiplicity	Intersection
-2	1	straight
1	2	tangent or bounce

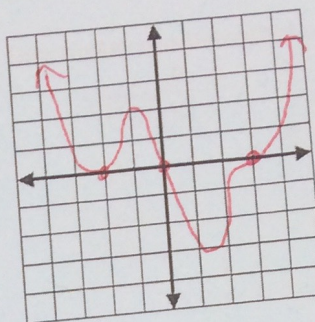
End Behavior: Deg: 3 Lc: + ↓↑

As $x \rightarrow -\infty, y \rightarrow -\infty$
 As $x \rightarrow +\infty, y \rightarrow +\infty$



10. $f(x) = x(x+2)^2(x-3)^3$

Zeros	Multiplicity	Intersection
0	1	straight
-2	2	tangent or bounce
3	3	inflection

End Behavior: Deg: 6 $LC: + \uparrow \uparrow$ As $x \rightarrow -\infty, y \rightarrow +\infty$ As $x \rightarrow +\infty, y \rightarrow +\infty$ For the following functions, graph on your calculator, state the zeros and multiplicity, write in factored form and analyze.

11. $f(x) = x^3 - x^2 - 6x$

Zeros	Multiplicity	Intersection
0	1	straight
-2	1	straight
3	1	straight

Factored form:

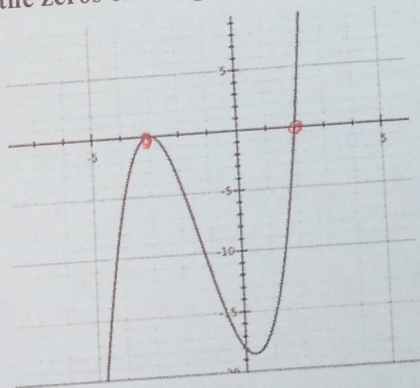
$$f(x) = x(x+2)(x-3)$$

End Behavior:

As $x \rightarrow -\infty, y \rightarrow -\infty$ As $x \rightarrow +\infty, y \rightarrow +\infty$

Find the zeros on the graph, then write the function in factored form.

13.



Zeros: -3 (mult. of 2), 2

Factored Form: $f(x) = (x+3)^2(x-2)$

12. $g(x) = x^4 - 17x^2 + 16$

Zeros	Multiplicity	Intersection
-4	1	straight
-1	1	straight
1	1	straight
4	1	straight

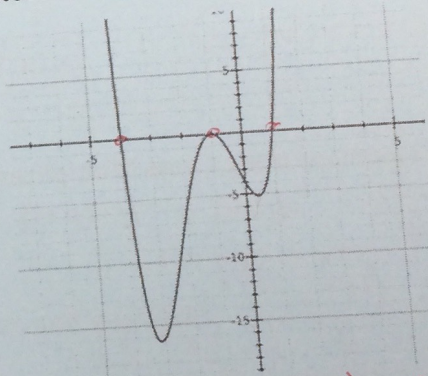
Factored form:

$$g(x) = (x+4)(x+1)(x-1)(x-4)$$

End Behavior:

As $x \rightarrow -\infty, y \rightarrow +\infty$ As $x \rightarrow +\infty, y \rightarrow +\infty$

14.



Zeros: -4, -1 (m. 2), 1

Factored Form: $f(x) = (x+4)(x+1)^2(x-1)$

Unit 4:
Perform the indicated operation and state the excluded values

1. $\frac{(x-5)(x+1)}{3x-15} \cdot \frac{4}{(x-3)(x+1)}$

$x \neq 5, 3, -1$

$\frac{4}{3(x-3)}$

2. $\frac{x+2}{x-4} \div \frac{x}{3x-12}$

$x \neq 4, 0$

$\frac{(x+2)}{(x-4)} \cdot \frac{3(x-4)}{x} = \frac{3(x+2)}{x}$

3. $\frac{x+3}{x+2} \div \frac{x(x+3)}{2x-4}$

$x \neq -2, 2, 0, -3$

$\frac{(x+3)}{(x+2)} \cdot \frac{2(x-2)}{x(x+3)} = \frac{2(x-2)}{x(x+2)}$

4. $\frac{3(x+2)}{3x+6} \cdot \frac{x-3}{x-4}$

$x \neq -2, 4$

$\frac{3(x+2)}{(x+2)} \cdot \frac{(x-3)}{(x-4)} = \frac{3(x-3)}{(x-4)}$

5. $\frac{6x+6}{(x+3)(x-3)} + \frac{(x+3)(x-3)}{(x+3)(x-3)}$

LCD: $(x+3)(x-3)$ $x \neq -3, 3$

$\frac{6x+6}{(x+3)(x-3)} + \frac{x^2-9}{(x+3)(x-3)} = \frac{x^2+6x-3}{(x+3)(x-3)}$

6. $\frac{4}{x-1} - \frac{(x+2)}{x-1}$

$x \neq 1$

$\frac{-x+2}{x-1}$

7. $\frac{4}{x(x-1)} - \frac{x+2}{x-1} \cdot \frac{x}{x}$

LCD: $x(x-1)$ $x \neq 0, 1$

$\frac{4}{x(x-1)} - \frac{x^2+2x}{x(x-1)}$

$\frac{-x^2-2x+4}{x(x-1)}$

Solve the equations using Cross Multiplication:

8. $\frac{3x}{7} = \frac{4x+1}{9}$

$9(3x) = 7(4x+1)$
 $27x = 28x + 7$
 $-28x = -28x$
 $-x = 7$
 $x = 7$

9. $\frac{x+5}{2} = \frac{-5}{x-6}$

$x \neq 6$

$(x+5)(x-6) = 2(-5)$
 $x^2 - 6x + 5x - 30 = -10$
 $+10$

$x^2 - x - 20 = 0$
 $(x-5)(x+4) = 0$

$x = 5, -4$

Solve the equation using "Kill the Denominator:"

10. $\frac{x}{x-1} + \frac{1}{x-3} = \frac{2}{(x-1)(x-3)}$

LCD: $(x-1)(x-3)$ $x \neq 1, 3$

$\frac{(x-1)(x-3)x}{(x-1)(x-3)} + \frac{(x-1)(x-3)}{(x-1)(x-3)} = \frac{2(x-1)(x-3)}{(x-1)(x-3)}$

$x^2 - 3x + x - 1 = \frac{2}{-2}$
 $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1$
 $x = 3$

11. $\frac{3}{x+2} + \frac{3}{2} = \frac{x}{2(x+2)}$

LCD: $2(x+2)$ $x \neq -2$

$\frac{2(x+2) \cdot 3}{(x+2)} + \frac{2(x+2) \cdot 3}{2} = \frac{2(x+2) \cdot x}{2(x+2)}$

$6 + 3x + 6 = x$
 $3x + 12 = x$
 $\frac{-x}{-x}$

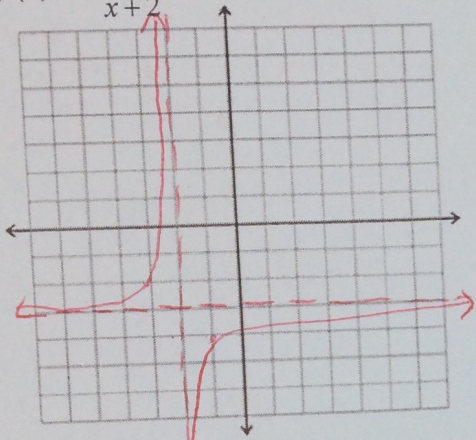
$2x + 12 = 0$
 $-12 -12$
 $2x = -24$
 $\frac{-24}{2} = -12$

$x = -6$

Unit 5:

List the transformations and graph the following:

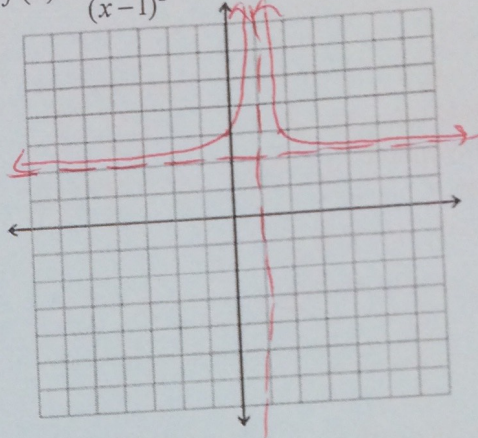
1. $f(x) = -\frac{1}{x+2} - 3$



Transformations:

- V. Flip
- Shift Left 2
- Shift Down 3

2. $f(x) = \frac{1}{(x-1)^2} + 2$



Transformations:

- Shift Right 1
- Shift Up 2

Use division to put into transformation form, then list the transformations from $f(x) = \frac{1}{x}$

3. $g(x) = \frac{3x-1}{x+2}$

$$\begin{array}{r} -2 \overline{) 3 \ -1} \\ \underline{3 \ -6} \\ \ 5 \end{array}$$

$$g(x) = \frac{-7}{x+2} + 3$$

- V. Stretch by -7 (V. Flip)
- Shift Left 2
- Shift Up 3

4. $g(x) = \frac{2x-3}{x-2}$

$$\begin{array}{r} 2 \overline{) 2 \ -3} \\ \underline{2 \ -4} \\ \ 1 \end{array}$$

$$g(x) = \frac{1}{x-2} + 2$$

- Shift Right 2
- Shift Up 2

For the following functions, identify the holes, asymptotes, and intercepts.

5. $f(x) = \frac{(x+4)(x-5)}{(x-5)(x+1)}$

Holes: $x=5$ V.A.: $x=-1$ H.A.: $y=1$ x-int: $(-4, 0)$ y-int: $(0, 4)$

6. $f(x) = \frac{x-3}{(x+1)(x+5)}$

Holes: None

V.A.: $x=-1, x=-5$ H.A.: $y=0$ x-int: $(3, 0)$ y-int: $(0, -3/5)$

Find the information for the functions, then graph.

$$7. f(x) = \frac{3x-6}{2x+4}$$

$$\text{Domain: } (-\infty, -2) \cup (-2, \infty)$$

$$\text{Range: } (-\infty, 3/2) \cup (3/2, \infty)$$

$$\text{x-int: } (2, 0)$$

$$\text{y-int: } (0, -3/2)$$

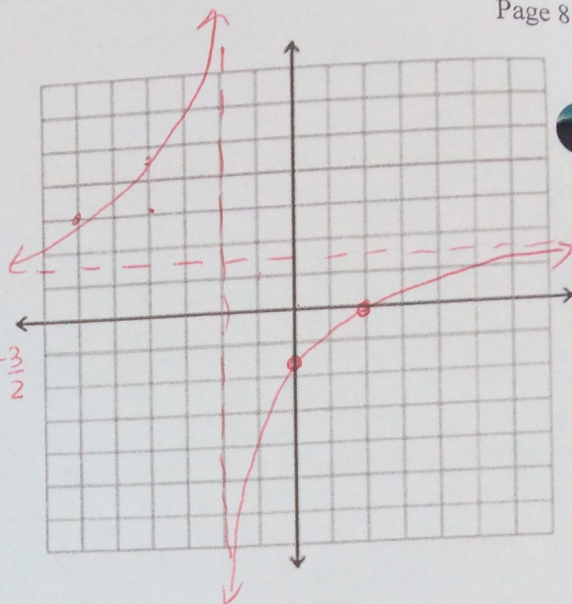
$$\text{Vertical Asymptote: } x = -2$$

$$\text{Horizontal Asymptote: } y = 3/2$$

End Behavior:

$$\text{As } x \rightarrow -\infty, y \rightarrow 3/2$$

$$\text{As } x \rightarrow +\infty, y \rightarrow 3/2$$



$$8. f(x) = \frac{(x+2)(x-3)}{(x-3)(x+1)}$$

$$\text{Domain: } (-\infty, -1) \cup (-1, 3) \cup (3, \infty)$$

$$\text{Range: } (-\infty, 1) \cup (1, \infty)$$

$$\text{x-int: } (-2, 0)$$

$$\text{y-int: } (0, 2)$$

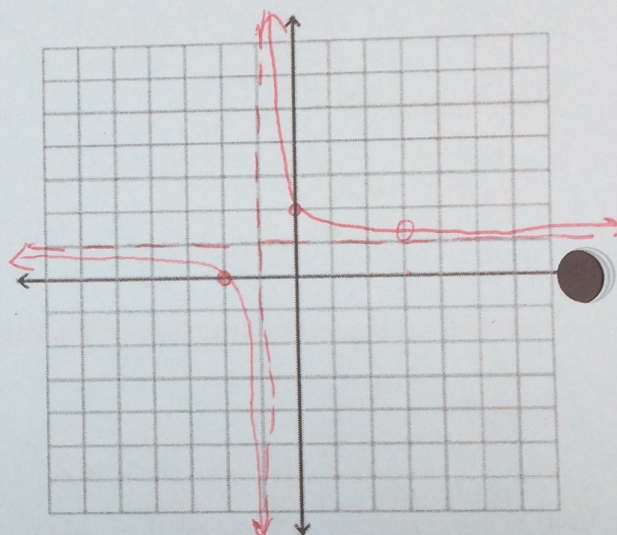
$$\text{Vertical Asymptote: } x = -1$$

$$\text{Horizontal Asymptote: } y = 1$$

End Behavior:

$$\text{As } x \rightarrow -\infty, y \rightarrow 1$$

$$\text{As } x \rightarrow +\infty, y \rightarrow 1$$



$$9. f(x) = \frac{10}{x-5}$$

$$\text{Domain: } (-\infty, 5) \cup (5, \infty)$$

$$\text{Range: } (-\infty, 0) \cup (0, \infty)$$

$$\text{x-int: None}$$

$$\text{y-int: } (0, -2)$$

$$\text{Vertical Asymptote: } x = 5$$

$$\text{Horizontal Asymptote: } y = 0$$

End Behavior:

$$\text{As } x \rightarrow -\infty, y \rightarrow 0$$

$$\text{As } x \rightarrow +\infty, y \rightarrow 0$$

