

1. Are $(x+2)$ and $(x-6)$ factors of $f(x) = 2x^3 + 8x^2 - 22x - 60$?

$$\begin{array}{r} -2 \overline{) 2 \ 8 \ -22 \ -60} \\ + \downarrow \ 4 \ \ 8 \ \ 60 \\ \hline 2 \ 4 \ -30 \ \ 0 \end{array}$$

$$\begin{array}{r} 6 \overline{) 2 \ 8 \ -22 \ -60} \\ + \downarrow \ 12 \ \ 120 \ \ 588 \\ \hline 2 \ 20 \ -98 \ \ 528 \end{array}$$

$$\begin{array}{r} 4 \\ \times 6 \\ \hline 24 \end{array}$$

$(x+2)$ is a factor. $(x-6)$ is not a factor.

Find **all** the zeros of the following functions

2. $g(x) = x^3 + 4x^2 + 4x$

$$\begin{aligned} & x(x^2 + 4x) \\ & x(x^2 + 4x + 4) \\ & x(x+2)(x+2) \end{aligned}$$

$x = 0, -2$
↑
m2

3. $h(x) = (3x^3 - 2x^2)(-3x + 2)$

$$\begin{aligned} & x^2(3x-2) - 1(3x-2) \\ & (3x-2)(x^2-1) \\ & (3x-2)(x+1)(x-1) \end{aligned}$$

$x = 2/3, -1, 1$

$$\begin{array}{r} 3x - 2 = 0 \\ +2 \quad +2 \\ \hline 3x = 2 \\ \hline x = 2/3 \end{array}$$

4. $f(x) = x^4 + x^3 - 14x^2 - 2x + 24$

Possible RP
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$$\begin{array}{r} 3 \overline{) 1 \ 1 \ -14 \ -2 \ 24} \\ + \downarrow \ 3 \ \ 12 \ \ -6 \ \ -24 \\ \hline 1 \ x^3 + 4x^2 - 2x - 8 \ \ 0 \end{array}$$

$$\begin{aligned} & x^2(x+4) - 2(x+4) \\ & (x+4)(x^2-2) \end{aligned}$$

$$\begin{array}{r} x^2 - 2 = 0 \\ +2 \quad +2 \\ \hline x^2 = 2 \\ \hline x = \pm\sqrt{2} \end{array}$$

5. $k(x) = (7x^3 + x^2)(-28x - 4)$

$$\begin{aligned} & x^2(7x+1) - 4(7x+1) \\ & (7x+1)(x^2-4) \\ & (7x+1)(x+2)(x-2) \end{aligned}$$

$x = -1/7, -2, 2$

$$\begin{array}{r} 7x + 1 = 0 \\ -1 \quad -1 \\ \hline 7x = -1 \\ \hline x = -1/7 \end{array}$$

Given the following zeros and multiplicities, write a function in factored form

6. 2 (multiplicity of 3), 5, -7 (multiplicity of 2)

7. 4, 2 (multiplicity of 5), -3

~~$(x-2)^3(x+5)$~~

$(x-2)^3(x-5)(x+7)^2$

$(x-4)(x-2)^5(x+3)$

8. Given $g(x) = 3x^3 - 8x^2 + 3x + 2$, use the rational root theorem to determine which of the following are possible zeros of the function.

\pm factors of const
factors of l.c.

$\pm 1, \pm 2, \pm 1/3, \pm 2/3$

a. 2

b. -3

c. 4

d. $2/3$

e. $3/4$

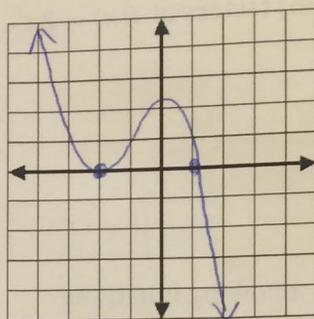
For the following functions, find the zeros, state the end behavior using limit notation, and graph the function.

9. $f(x) = -(x+2)^2(x-1)$

zeros: $-2, 1$
End Behavior: $\uparrow\downarrow$

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = -\infty$

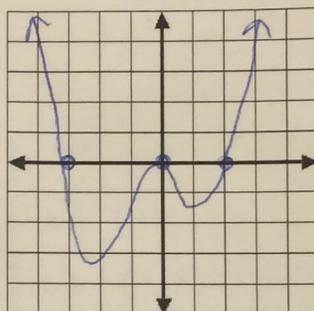


10. $h(x) = x(x+3)^2(x-2)^3$

zeros: $0, -3, 2$
End Behavior: $\uparrow\uparrow$

$\lim_{x \rightarrow \infty} f(x) = \infty$

$\lim_{x \rightarrow -\infty} f(x) = \infty$



11. $f(x) = x^3 - 10x^2 + 14x + 16$

Possible PR
 $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrr} 8 & 1 & -10 & 14 & 16 \\ & + & \downarrow & & \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

$x^2 - 2x - 2$

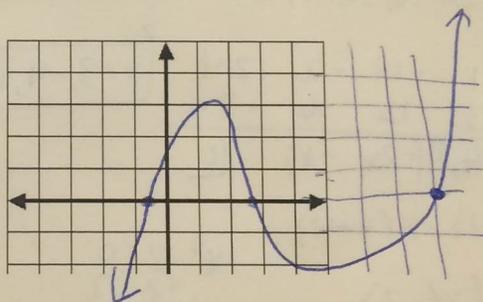
$$\frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

zeros: $8, 1+\sqrt{3}, 1-\sqrt{3}$

End Behavior: $\downarrow\uparrow$ or $\uparrow\downarrow$

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$



12. $g(x) = x^4 - 17x^2 + 16$

$(x^2 - 16)(x^2 - 1)$

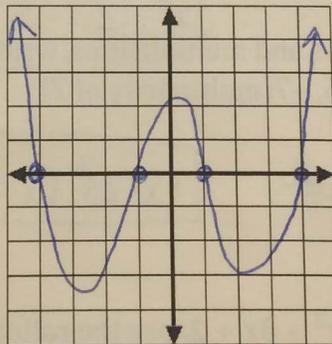
$(x+4)(x-4)(x+1)(x-1)$

zeros: $-4, 4, -1, 1$

End Behavior: $\uparrow\uparrow$

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

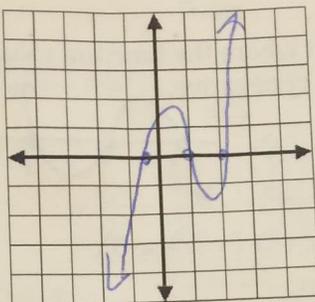


3. $f(x) = 3x^3 - 8x^2 + 3x + 2$

Possible RR

$\pm 1, \pm 2, \pm 1/3, \pm 2/3$

$$\begin{array}{r} \downarrow \cdot 3 \quad -8 \quad 3 \quad 2 \\ + \downarrow \quad 3 \quad -5 \quad -2 \\ \hline 3x^2 - 5x - 2 \quad | 0 \\ (3x+1)(x-2) \end{array}$$



zeros: $x = 1, -1/3, 2$

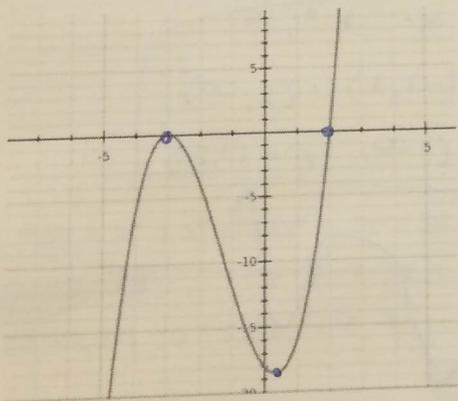
End Behavior: $\downarrow \uparrow$

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

Given the following graphs analyze the functions

14.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

#/type max: $(-3, 0)$

#/type min: $(-0.5, -18)$

x-intercept(s): $(-3, 0), (2, 0)$

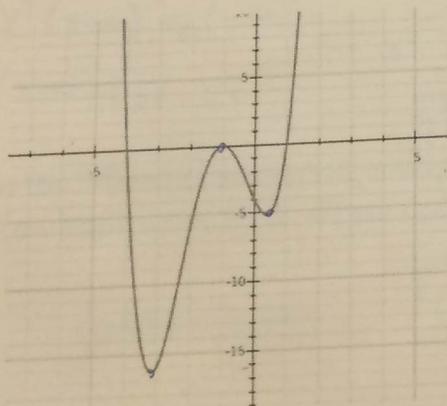
y-intercept: ~~(0, -18)~~ $(-18, 0)$

End Behavior:

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

5.



Domain: $(-\infty, \infty)$

Range: $[-16, \infty)$

#/type max: $(-1, 0)$

#/type min: $(-3, -16), (0.5, -5)$

x-intercept(s): $(-4, 0), (-1, 0), (1, 0)$

y-intercept: $(0, -4)$

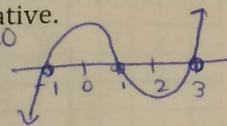
End Behaviors:

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

Determine the intervals where the function values are
 a) zero, b) positive and c) negative.

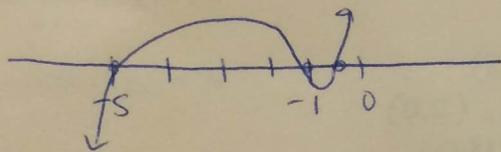
16. $f(x) = (x^3 - 3x^2)(-x + 3)$
 $x^2(x-3) \cdot (-1)(x-3)$
 $(x-3)(x^2-1) = (x-3)(x+1)(x-1)$



- a) $x = 3, -1, 1$
- b) $(1, 3) \cup (3, \infty)$
- c) $(-\infty, -1) \cup (1, 3)$

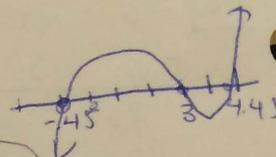
18. $h(x) = 2x^3 + 13x^2 + 16x + 5$

- a) $x = -5, -1, -1/2$
- b) $(-5, -1) \cup (-1/2, \infty)$
- c) $(-\infty, -5) \cup (-1, -1/2)$



17. $g(x) = x^3 - 7x^2 + 10x + 6$

- a) $x = -0.45, 3, 4.45$
- b) $(-0.45, 3) \cup (4.45, \infty)$
- c) $(-\infty, -0.45) \cup (3, 4.45)$



19. $f(x) = (x^3 - 2x^2)(x + 2)$

$x^2(x-2) \cdot 1(x+2)$
 $(x^2-1)(x-2)$
 $(x+1)(x-1)(x-2)$

- a) $x = -1, 1, 2$
- b) $(-1, 1) \cup (2, \infty)$
- c) $(-\infty, -1) \cup (1, 2)$

